Chair of Network Architectures and Services
Department of Informatics
Technical University of Munich

Note:
- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Network Coding

Exam: IN2315 / Endterm
Examiner: Prof. Dr.-Ing. Georg Carle
Date: Wednesday 14th February, 2018
Time: 10:30 – 11:45

	P 1	P 2	P 3	P 4
I				
II				

Working instructions

- This exam consists of
  - **16 pages** with a total of **4 problems** and
  - a two-sided printed **cheat sheet**.

  Please make sure now that you received a complete copy of the exam.
- Detaching pages from the exam is prohibited.
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- The total amount of achievable credits in this exam is 60 credits.
- Allowed resources:
  - one **analog dictionary** English ↔ native language
  - one **self-written cheat sheet** (A4 double-sided)
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________ to ________ / Early submission at ________

Problem 1 IEEE 802.11 wireless networks (11 credits)

In this problem we consider an ordinary IEEE 802.11-based network as depicted in Figure 1.1. The two wireless devices and the access point are operating in infrastructure mode. The access points connects the wireless network to a Ethernet-based local network. The whole network makes up a single subnet.

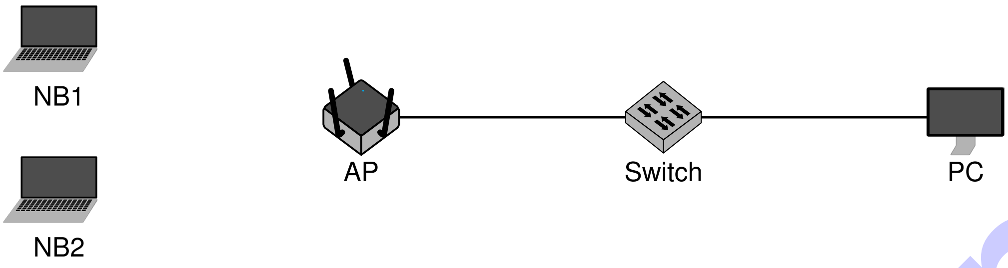


Figure 1.1: IEEE 802.11-based wireless network

0 1 2

a)* What is the difference between *collision detection* and *collision avoidance* with respect to medium access?

Collision detection assumes that a transmission was successfull if no collision was detected during transmission.
Collision avoidance tries to minimize the probability of collisions by randomizing the medium access.

0 1 2

b)* Explain two major differences between Ethernet header and the (generic) IEEE 802.11 header.

In addition to transmitter and receiver address, the IEEE 802.11 header may also provide source and destination MAC addresses.
The frame control field of the IEEE 802.11 allows to differentiate between the various frame types and subtypes, which are not needed in Ethernet.

c)* Name the three major frame types used by IEEE 802.11 and give one example for each type.

0 1 2 3

Data frames (e. g. subtype data or QoS), management frames (e. g. beacons, association requests, etc.), and control frames (e. g. acknowledgements, RTS/CTS, etc.).

d)* Explain how a frame from NB1 to NB2 is being transmitted.

0 1 2

The frame is first sent to the AP (transmitter NB1, reiceiver AP, destination NB2), and than to NB (transmitter AP, source NB1, receiver NB2).

e)* Assuming that NB1 wants to communicate with PC. State the MAC addresses of the frame

1. between NB1 and the AP, and
2. between the AP and PC.

0 1 2

Hint: You may simply write a node's name as its MAC address, e. g. NB1 to denote the MAC address of node NB1.

1. transmitter NB1, receiver AP, destination PC
2. transmitter NB1, receiver PC

Problem 2 Finite extension fields (14 credits)

Given the finite field $\mathbb{F}_p$, we consider the finite extension field

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \mid a_i \in \mathbb{F}_p \right\}. \tag{1}$$

a)* State the conditions on p, q, and n such that a finite extension field $F_q[x]$ exists.

$q = p^n$ with $p, n \in \mathbb{N}$ and p prime.

We now consider the specific extension field $F_q[x]$ resulting from $p = 5$ and $n = 2$.

b)* Find a generator of $\mathbb{F}_5$ and give a proof for your choice.

$2 \in \mathbb{F}_5$ is a generator: (all operations mod 5) $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 3$, $2^4 = 1$

c)* List all elements of $F_q[x]$.

$$\begin{aligned} F_q[x] = \{ & 0, 1, 2, 3, 4, \\ & x, x+1, x+2, x+3, x+4, \\ & 2x, 2x+1, 2x+2, 2x+3, 2x+4, \\ & 3x, 3x+1, 3x+2, 3x+3, 3x+4, \\ & 4x, 4x+1, 4x+2, 4x+3, 4x+4 \ \} \end{aligned}$$

d)* Explain the purpose of a *reduction polynomial*.

Given two elements $a, b \in F_q[x]$, the product $a \cdot b$ is in general not an element of $F_q[x]$. The reduction polynomial ensures that the result is in turn an element of $F_q[x]$.

e)* State the conditions a reduction polynomial has to fullfill.

It must be of degree n and must not be the product of two elements from $F_q[x]$.

f) Find a reduction polynomial for $F_q[x]$ and give a proof for your choice.

The reduction polynomial must be of degree 2 and, therefore, the factors must both be of degree 1. Furthermore, it is sufficient to check all polynomials with the highest order monom of x^2 (coefficient is 1) as any other reduction polynomial not found that way is only a multiple of those polynomials.

$$\begin{aligned}
x \cdot x &= x^2 \\
x(x+1) &= x^2 + x \\
x(x+2) &= x^2 + 2x \\
x(x+3) &= x^2 + 3x \\
x(x+4) &= x^2 + 4x \\
(x+1)^2 &= x^2 + 2x + 1 \\
(x+1)(x+2) &= x^2 + 3x + 2 \\
(x+1)(x+3) &= x^2 + 4x + 3 \\
(x+1)(x+4) &= x^2 + 4 \\
(x+2)^2 &= x^2 + 4x + 4 \\
(x+2)(x+3) &= x^2 + 1 \\
(x+2)(x+4) &= x^2 + x + 3 \\
(x+3)^2 &= x^2 + x + 4 \\
(x+3)(x+4) &= x^2 + 2x + 2 \\
(x+4)^2 &= x^2 + 3x + 1
\end{aligned}$$

Possible reduction polynomials are therefore

$$\begin{aligned}
&x^2 + 2, \\
&x^2 + 3, \\
&x^2 + x + 1, \\
&x^2 + x + 2, \\
&x^2 + 2x + 3, \\
&x^2 + 2x + 4, \\
&x^2 + 3x + 3, \\
&x^2 + 3x + 4, \\
&x^2 + 4x + 1, \\
&x^2 + 4x + 2,
\end{aligned}$$

as well as any of those polynomials multiplied by an element from $\mathbb{F}_5 \backslash \{0\}$.

Problem 3 Reformulating optimization problems (8 credits)

We consider the max-flow problem on a hypergraph $G = (V, H)$ in the lossy hyperarc model as stated in (2). As introduced in class and tutorials, $\boldsymbol{x}$ denotes the flow vector, $\boldsymbol{M}$ the incidence matrix describing the induced graph, $\boldsymbol{d}$ the demand vector, $\boldsymbol{N}$ the hyperarc-arc incidence matrix, $\boldsymbol{y}$ the broadcast capacity vector, and $\mathcal{Y}$ the broadcast capacity region.

$$\begin{aligned} \max r \quad \text{s.t.} \quad \boldsymbol{x} &\geq \boldsymbol{0} \\ \boldsymbol{Mx} &= r\boldsymbol{d} \\ \boldsymbol{Nx} &\leq \boldsymbol{y} \\ \boldsymbol{y} &\in \mathcal{Y} \end{aligned} \tag{2}$$

We want to reformulate this optimization problem to a form suitable for Matlab:

$$\begin{aligned} \min \boldsymbol{f}^T\boldsymbol{\xi} \quad \text{s.t.} \quad \boldsymbol{A}\boldsymbol{\xi} &\leq \boldsymbol{b} \\ \boldsymbol{A}'\boldsymbol{\xi} &= \boldsymbol{b}' \\ \boldsymbol{c} \leq \boldsymbol{\xi} &\leq \boldsymbol{c}' \end{aligned} \tag{3}$$

The $\in$ (element of) relation cannot directly be represented by the canonical form given in (3). We therefore introduce the vector $\boldsymbol{\tau}$ of time shares as well as the constraints $\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$ and $\boldsymbol{\tau} \geq \boldsymbol{0}$, which are implied by the broadcast capacity region $\mathcal{Y}$.

a)* Find a matrix $\boldsymbol{Y} \in \mathbb{R}^{|H| \times |V|}$ such that $\boldsymbol{y} \leq \boldsymbol{Y\tau}$ becomes an equivalent constraint to $\boldsymbol{y} \in \mathcal{Y}$.
Hint: Elements Y_{jv} of $\boldsymbol{Y}$ represent probabilities.

$$Y_{jv} = \begin{cases} 1 - \prod_{k \in A_j} \epsilon_k \text{ or } y_j/\tau_v & \text{if } (v, B) \equiv j \\ 0 & \text{otherwise} \end{cases}$$

For the reformulation we collect all optimization variables into a single vector, which is defined as $\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{\tau} \\ r \end{bmatrix}$.

b)* Rewrite the equality constraint $\boldsymbol{Mx} = r\boldsymbol{d}$ to the form $[...]\boldsymbol{\xi} = \boldsymbol{0}$.

$$\begin{aligned} \boldsymbol{Mx} &= r\boldsymbol{d} \\ \boldsymbol{Mx} - r\boldsymbol{d} &= \boldsymbol{0} \\ [\boldsymbol{M} \quad \boldsymbol{0} \quad -\boldsymbol{d}]\boldsymbol{\xi} &= \boldsymbol{0} \end{aligned}$$

c)* Rewrite the inequality constraint $\boldsymbol{N}\boldsymbol{x} \leq \boldsymbol{Y}\boldsymbol{\tau}$ to the form $[...]\boldsymbol{\xi} \leq \boldsymbol{0}$.

$$\boldsymbol{N}\boldsymbol{x} \leq \boldsymbol{Y}\boldsymbol{\tau}$$
$$\boldsymbol{N}\boldsymbol{x} - \boldsymbol{Y}\boldsymbol{\tau} \leq \boldsymbol{0}$$
$$[\boldsymbol{N} \quad -\boldsymbol{Y} \quad \boldsymbol{0}]\boldsymbol{\xi} \leq \boldsymbol{0}$$

d)* Rewrite the inequality constraint $\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$ to the form $[...]\boldsymbol{\xi} \leq 1$.

$$\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$$
$$[\boldsymbol{0} \quad \boldsymbol{1}^T \quad 0]\boldsymbol{\xi} \leq 1$$

e) Combine the results from Subproblems b) – d) to the canonical flow problem as given in (3).

$$\min \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ -1 \end{bmatrix}^T \boldsymbol{\xi} \qquad \text{s. t.} \begin{bmatrix} \boldsymbol{N} & -\boldsymbol{Y} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1}^T & 0 \end{bmatrix} \boldsymbol{\xi} \leq \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} & -\boldsymbol{d} \end{bmatrix} \boldsymbol{\xi} = \boldsymbol{0}$$

$$\boldsymbol{\xi} \geq \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ 0 \end{bmatrix}$$

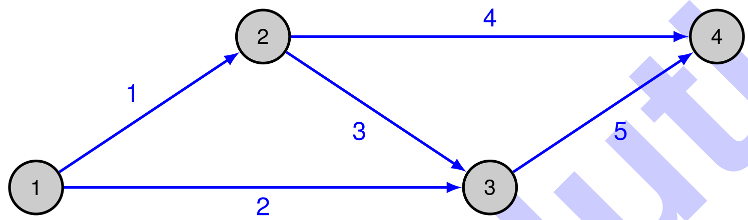
Problem 4 Network coding in lossy wireless packet networks (27 credits)

We consider the network represented as induced graph $G = (N, A)$ as defined by its incidence matrix

$$\boldsymbol{M} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}. \tag{4}$$

We assume that packet losses, i. e., erasure events, are indepedently and identically distributed with expectation ε_k for all $k \in A$. Assume that all arcs $k \in A$ have unit capacity. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume orthogonal medium access, i. e., nodes do not transmit concurrently.

a) Draw the induced graph $G = (N, \mathcal{A})$ and assign indices $k \in A$ to all arcs in lexicographic order.
Hint: An additional preprint can be found in Figure 4.1.



b)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic ascending order and assign numbers $j \equiv (a, B)$ in Table 4.1.

c)* List the set of induced arcs A_j for all $j \in H$ in Table 4.1.

d)* Determine the network's hyperarc capacity region (Table 4.1).

e) Determine the network's broadcast capacity region (Table 4.1).

We now consider a unicast session between nodes 1 and 4.

f)* List all $s - t$ cuts.

$S_1 = \{1\}$, $S_2 = \{1, 2\}$, $S_3 = \{1, 3\}$, $S_4 = \{1, 2, 3\}$

g) Determine the value of each $s - t$ cut.

$$v(S_1) = y_3 = \tau_1(1 - \varepsilon_1\varepsilon_2)$$
$$v(S_2) = y_2 + y_6 = \tau_1(1 - \varepsilon_2) + \tau_2(1 - \varepsilon_3\varepsilon_4)$$
$$v(S_3) = y_1 + y_7 = \tau_1(1 - \varepsilon_1) + \tau_3(1 - \varepsilon_5)$$
$$v(S_4) = y_5 + y_7 = \tau_2(1 - \varepsilon_4) + \tau_3(1 - \varepsilon_5)$$

$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	A_j	z_j	y_j
$(1, \{2\})$	1	$\{(1, 2)\}$	$\tau_1(1 - \varepsilon_1)\varepsilon_2$	$\tau_1(1 - \varepsilon_1)$
$(1, \{3\})$	2	$\{(1, 3)\}$	$\tau_1(1 - \varepsilon_2)\varepsilon_1$	$\tau_1(1 - \varepsilon_2)$
$(1, \{2, 3\})$	3	$\{(1, 2), (1, 3)\}$	$\tau_1(1 - \varepsilon_1)(1 - \varepsilon_2)$	$\tau_1(1 - \varepsilon_1\varepsilon_2)$
$(2, \{3\})$	4	$\{(2, 3)\}$	$\tau_2(1 - \varepsilon_3)\varepsilon_4$	$\tau_2(1 - \varepsilon_3)$
$(2, \{4\})$	5	$\{(2, 4)\}$	$\tau_2(1 - \varepsilon_4)\varepsilon_3$	$\tau_2(1 - \varepsilon_4)$
$(2, \{3, 4\})$	6	$\{(2, 3), (2, 4)\}$	$\tau_2(1 - \varepsilon_3)(1 - \varepsilon_4)$	$\tau_2(1 - \varepsilon_3\varepsilon_4)$
$(3, \{4\})$	7	$\{(3, 4)\}$	$\tau_3(1 - \varepsilon_5)$	$\tau_3(1 - \varepsilon_5)$

Table 4.1: Fill in values from different subproblems. (An additional preprint can be found in Table 4.2)

For some specific ε_k, the cut capacities become

$$v(S_1) = \frac{3}{4}\tau_1, \; v(S_2) = \frac{1}{2}\tau_1 + \frac{3}{4}\tau_2, \; v(S_3) = \frac{1}{2}\tau_1 + \frac{7}{8}\tau_3, \text{ and } v(S_4) = \frac{1}{2}\tau_2 + \frac{7}{8}\tau_3.$$

h)* What is the effect on $v(S_3)$ and $v(S_4)$ if τ_1 and τ_2 are being increased?

Both cut values decrease since increases τ_1 and τ_2 implies decreasing τ_3 and both cuts benefit most from τ_3.

i)* Which condition must hold for τ_i with $i \in N$?

$$\sum_{i \in N} \tau_i = 1$$

j)* Assuming that $v(S_1) = v(S_2)$ must hold for an optimal solution, determine τ_1, τ_2, and τ_3.

- From $v(S_1) = v(S_2)$ we obtain that $\tau_2 = {}^1\!/\!_3\tau_1$.
- Since $v(S_4)$ is smaller than $v(S_3)$, the latter one is not binding.
- Setting $v(S_1) = v(S_4)$ yields $\tau_3 = {}^2\!/\!_3\tau_1$.
- With $\tau_1 + \tau_2 + \tau_3 = 1$ we finalle obtain $\tau_1 = {}^1\!/\!_2$, $\tau_2 = {}^1\!/\!_6$, and $\tau_3 = {}^1\!/\!_3$.

The correct solution of the previous subroblem is $\tau_1 = 1/2$, $\tau_2 = 1/6$, and $\tau_3 = 1/3$. (Don't even think about using these results to find the correct solution.)

k)* Show that this solution is indeed optimal.

0
1
2
3
4

- $v(S_1) = v(S_2) = v(S_4) = 3/8$ and $v(S_3) = 13/24$.
- To obtain a better solution we would have to must increase the capacity of S_1, S_2, and S_4 concurrently.
- In order to increase the capacity of S_1 and S_2 we have to increase τ_1 but must not decrease τ_2 because this would decrease the capacity of S_2.
- Consequently τ_3 must be decreased, which however decreases the capacity of S_4 which is also binding.

$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	A_j	z_j	y_j
$(1, \{2\})$	1	$\{(1, 2)\}$	$\tau_1(1 - \varepsilon_1)\varepsilon_2$	$\tau_1(1 - \varepsilon_1)$
$(1, \{3\})$	2	$\{(1, 3)\}$	$\tau_1(1 - \varepsilon_2)\varepsilon_1$	$\tau_1(1 - \varepsilon_2)$
$(1, \{2, 3\})$	3	$\{(1, 2), (1, 3)\}$	$\tau_1(1 - \varepsilon_1)(1 - \varepsilon_2)$	$\tau_1(1 - \varepsilon_1\varepsilon_2)$
$(2, \{3\})$	4	$\{(2, 3)\}$	$\tau_2(1 - \varepsilon_3)\varepsilon_4$	$\tau_2(1 - \varepsilon_3)$
$(2, \{4\})$	5	$\{(2, 4)\}$	$\tau_2(1 - \varepsilon_4)\varepsilon_3$	$\tau_2(1 - \varepsilon_4)$
$(2, \{3, 4\})$	6	$\{(2, 3), (2, 4)\}$	$\tau_2(1 - \varepsilon_3)(1 - \varepsilon_4)$	$\tau_2(1 - \varepsilon_3\varepsilon_4)$
$(3, \{4\})$	7	$\{(3, 4)\}$	$\tau_3(1 - \varepsilon_5)$	$\tau_3(1 - \varepsilon_5)$

Table 4.2: Additional preprint for Table 4.1

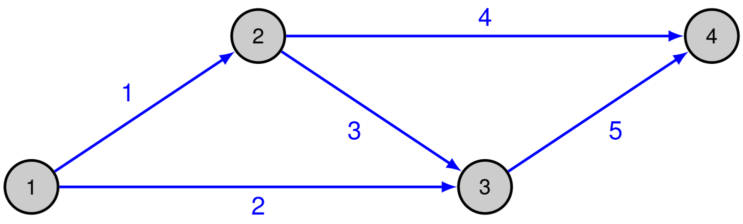


Figure 4.1: Additional preprint for Problem 4 a)

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.