Chair of Network Architectures and Services
Department of Informatics
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Esolution
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Note:
- During the attendance check a sticker containing a unique code will be put on this exam.
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Network Coding

Exam: IN2315 / Endterm
Examiner: Prof. Dr.-Ing. Georg Carle
Date: Friday 22nd February, 2019
Time: 13:30 – 15:00

	P 1	P 2	P 3	P 4
I				
II				

Working instructions

- This exam consists of
  - **16 pages** with a total of **4 problems** and
  - a two-sided printed **cheat sheet**.

  Please make sure now that you received a complete copy of the exam.
- Detaching pages from the exam is prohibited.
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- The total amount of achievable credits in this exam is 60 credits.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
  - one **self-written cheat sheet** (A4 double-sided)
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ___________ to ___________ / Early submission at ___________

Problem 1 Link quality estimation and estimation of redundancy (10 credits)

In this problem we start with the well-known ETX metric introduced in the lecture and develop advanced link quality estimators. We limit the problem to a two-node wireless network with symmetric channel conditions. For sake of simplicity we assume independent and uniformely distrubted packet loss with probability $0 < \epsilon < 1$. Throughout the problem you may assume a generation size of $N > 0$ packets being transmitted from one node to the other.

0 1

1.1* State the number n' of frames to be transferred from one node to the other according to the ETX metric.

$$n' = \frac{N}{1-\epsilon}$$

0 1 2

1.2* Explain the problem arising from transmitting n' frames, in particular regarding link establishment such as ARP request / response or TCP-handshakes.

There is a considerable probability of more than 40 % that transmitting n' frames is insufficient. That delays link establishement as protocols have to wait for timeout conditions for retransmits to occurr.

0 1 2

1.3* Determine the probability that transmitting n packets allows to decode a generation of N. You may assume that there are no random linear dependencies in transmitted packets.

Let X denote the random variable indicating the number of linear independent packets received.

$$\Pr[X \geq N \mid n] = \begin{cases} 0 & n < N, \\ \sum_{i=N}^{n} \binom{n}{i} (1-\epsilon)^i \epsilon^{n-1} & \text{otherwise.} \end{cases}$$

In order to remedy the situation in Subproblem 1.2, we seek a metric that allows to specify a confidence factor $0 < \sigma < 1$ that transmitting n frames results in a decoding probability of at least σ.

0 1

1.4 Based on Subproblem 1.3, state the optimization problem that determines the minimum number n^* of packets required to achieve a decoding probability of at least σ.

Let X denote the random variable indicating the number of linear independent packets received.

$$\min_{n \geq N} \quad \text{s.t. } \Pr[X \geq N \mid n] \geq \sigma$$

1.5 Assuming we choose $\sigma = 0.99$, briefly discuss the effect on a link establishment phase as described above, and on a bulk data transfer.

0 1 2

$\sigma = 0.99$ is very high, resulting in a large amount of superfluously transmitted packets. That is acceptible during delay-sensitive phases of low traffic , but results in an intolerable amount of redundant packets in case of a bulk data transfer.

1.6 In the lecture we introduced RALQE. Discuss how it prevents the situation of Subproblem 1.5.

0 1 2

It takes the reliability of the underlying link quality estimator into account .
During phases of low traffic, few samples are available, leading to an unreliable estimator. In that case it affords to transmit more packets than needed. .
During bulk transfers, many samples are available, leading to a more relaible and stable estimation of the link quality, and in that case in converges to the optimal amount n^* of packets.

Problem 2 IEEE 802.11 medium access (19 credits)

This problem discusses the distributed coordination function (DCF), which is the basic medium access strategy of IEEE 802.11-based networks. The DCF is schematically depicted in Figure 2.1.

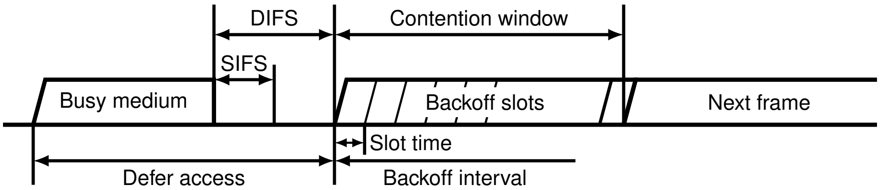


Figure 2.1: IEEE 802.11 medium access

2.1* Explain how the DCF works when a node is ready to transmit a frame (assuming no prior frame loss).

After a minimum idle time of DIFS + 2SIFS a random value from the contention window $\{0, 1, \ldots, N\}$ is drawn. Medium access is deferred for that amount of time slots.
If the medium is still idle after that time period, a transmission attempt is made in the following time slot.
If the medium becomes busy in the mean time, transmission and countdown are deferred until the medium becomes idle again.
In case of an unsuccessful transmission, the contention window is exponentially increased up to some maximum value.

2.2* How is frame loss detected in case of unicasts and multicasts?

Missing L2-Ack in case of unicasts, no detection possible for multicasts.

2.3* Explain whether or not transmitting nodes are able to differentiate between frame loss and collisions.

They are in gernal unable for two reasons:

1. A transmitting node is commonly not able to concurrently sense the medium.
2. Even if a node was able to do that, the second transmission involved into a collision near the receiving node might be out of range (hidden station problem).

2.4* Explain whether or not the DCF is fully functional in case of nodes operating in monitor mode.

0 1 2

No, it is not as nodes operating in monitor mode do not transmit L2-ACKs. Therefore, a transmitter is unable to adjust the binary exponential backoff as frame losses cannot be detected.

We now assume a network consisting of two nodes operating in monitor mode in range of each other. For the sake of simplicity we assume that

- both nodes are backlogged,
- no further communication of other nodes takes place,
- no random frame losses occur, and
- both nodes are perfectly synchronized, i. e., time is slotted and both nodes have a common view of when a time slot starts.

Let $X_i \in \{0, 1, \ldots, N_i\}$ denote the random variable denoting the number of contention slots drawn by node $i \in \{1, 2\}$.

2.5* Determine the expectation $\mathbb{E}[X_i]$ and briefly discuss its influence on the expected maximum throughput.

0 1 2

$\mathbb{E}[X_i] = N_i/2$ as X_i is drawn uniformely from the set $\{0, 1, \ldots, N_i\}$.
The larger $\mathbb{E}[X_i]$ becomes, the less the probability for a collision. However, more time is wasted for medium access resulting in lower maximum throughput.

2.6* Derive the probability of a collision in case of $N_1 = N_2 = N$.

0 1

$$\begin{aligned}\Pr[\text{“Collision”}] &= \sum_{k=0}^{N} \Pr[X_1 = X_2] \\ &= \sum_{k=0}^{N} \frac{1}{(N+1)^2} = \frac{1}{N+1}\end{aligned}$$

2.7* Derive the probability of a collision in case of $N_1 < N_2$.

$$\begin{aligned}
\Pr[\text{“Collision”}] &= \sum_{k=0}^{\min\{N_1,N_2\}} \Pr[X_1 = X_2] \\
&= \sum_{k=0}^{\min\{N_1,N_2\}} \frac{1}{(N_1+1)(N_2+1)} \\
&= \frac{N_1+1}{(N_1+1)(N_2+1)} = \frac{1}{N_2+1}
\end{aligned}$$

2.8* Derive the probability that node 2 successfully transmits a frame in that case.

$$\begin{aligned}
\Pr[\text{“Node 2 successfull”}] &= \Pr[X_2 < X_1] \\
&= \sum_{k=0}^{N_1} \Pr[X_2 = k] \Pr[X_1 > k] \\
&= \frac{1}{N_2+1} \sum_{k=0}^{N_1} (1 - \Pr[X_1 \leq k]) \\
&= \frac{1}{N_2+1} \sum_{k=0}^{N_1} \left(1 - \frac{k+1}{N_1+1}\right) \\
&= \frac{1}{N_2+1} \left(N_1 + 1 - \frac{1}{N_1+1} \sum_{k=1}^{N_1+1} k\right) \\
&= \frac{1}{N_2+1} \left(N_1 + 1 - \frac{1}{N_1+1} \frac{(N_1+1)(N_1+2)}{2}\right) \\
&= \frac{N_1}{2(N_2+1)}
\end{aligned}$$

Problem 3 Random Linear Network Coding (21 credits)

We consider extension fields of order $q = p^n$, i. e., sets of polynomials

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \mid a_i \in \mathbb{F}_p \right\}.$$

Elements of such an extension field can also be denoted in short form as $f(x) \in F_q[x] \equiv f(p)$, i. e.,

$$f(x) = x^2 + 2x + 1 \in F_{5^3}[x] \equiv f(5) = 5^2 + 2 \cdot 5 + 1 = 31. \tag{1}$$

3.1* Which conditions must p and n fulfill such that $F_q[x]$ becomes an extension field?

p must be prime and $n \in \mathbb{N}$ a natural number.

We now consider $p = 3$, i. e., we have coefficients $a_i \in \mathbb{F}_3$.

3.2* State the summation and multiplication tables for $\mathbb{F}_3$.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

We now construct an extension field of order $q = 9$, i. e., $p = 3$ and $n = 2$.

3.3* List all elements of $F_9[x]$.

$F_9[x] = \{0, 1, 2, x, x+1, x+2, 2x, 2x+1, 2x+2\}$

0 1 2 3 4

3.4 Find a reduction polynomial for $F_9[x]$.

It is sufficient to consider polynomials of the form $x^2 + \ldots$.

$$x \cdot x = x^2$$
$$x \cdot (x+1) = x^2 + x$$
$$x \cdot (x+2) = x^2 + 2x$$
$$(x+1) \cdot (x+1) = x^2 + 2x + 1$$
$$(x+1) \cdot (x+2) = x^2 + 2$$
$$(x+2) \cdot (x+2) = x^2 + x + 1$$

Possible reduction polynomials are therefore $x^2 + 1$, $x^2 + x + 2$ and $x^2 + 2x + 2$.

0 1 2 3 4 5 6

3.5 State the summation and multiplication tables for F_9.

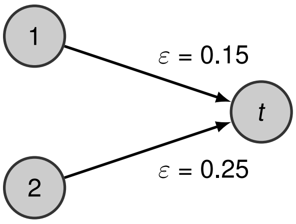
+	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	0	4	5	3	7	8	6
2	2	0	1	5	3	4	8	6	7
3	3	4	5	6	7	8	0	1	2
4	4	5	3	7	8	6	1	2	0
5	5	3	4	8	9	7	2	0	1
6	6	7	8	0	1	2	3	4	5
7	7	8	6	1	2	0	4	5	3
8	8	6	7	2	0	1	5	3	4

·	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	1	6	8	7	3	5	4
3	0	3	6	2	5	8	1	4	7
4	0	4	8	5	6	1	7	2	3
5	0	5	7	8	1	3	4	6	2
6	0	6	3	1	7	4	2	8	5
7	0	7	5	4	2	6	8	3	1
8	0	8	4	7	3	2	5	1	6

In Figure 3.1 we see a fragment of a network. Node t is the destination of some flow. The network coding uses the finite field F_9 with a generation size of 3. The coding vectors and (coded) packets known to each node are depicted next to the respective node.

$$\boldsymbol{C}_1 = \left[\begin{array}{ccc|cccc} 1 & 3 & 2 & 7 & 8 & 1 & 3 \\ 2 & 2 & 3 & 3 & 6 & 0 & 7 \end{array}\right]$$

$$\boldsymbol{C}_2 = \left[\begin{array}{ccc|cccc} 2 & 1 & 6 & 7 & 5 & 3 & 6 \\ 1 & 2 & 3 & 1 & 7 & 1 & 1 \end{array}\right]$$



$$\boldsymbol{C}_t = \left[\begin{array}{ccc|cccc} 1 & 2 & 3 & 5 & 7 & 6 & 3 \end{array}\right]$$

Figure 3.1: Fragment of a Network

Node t has already received one packet but still needs at least two more packets to be able to decode. We want to maximize the probability that an innovative packet is received when a packet is sent.

3.6 Argue which node should transmit a packet next.

Node 2 has only linear dependent coding vectors, node 1 has only independent coding vectors.
Node 1 has a lower packet error rate.
⇒ Node 1 should sent a packet.

3.7 Determine the probability that an innovative packet is received if node 2 transmits a packet.

As node 2 has only linear dependent coding vectors, the probability for an innovative packet is 0 %.

0 1 2 3 4

3.8 Assuming that two linear independent packets were received by node t, decode the packets.

$$\left[\begin{array}{ccc|cccc} 1 & 2 & 3 & 5 & 7 & 6 & 3 \\ 1 & 3 & 2 & 7 & 8 & 1 & 3 \\ 2 & 2 & 3 & 3 & 6 & 0 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|cccc} 1 & 2 & 3 & 5 & 7 & 6 & 3 \\ 0 & 8 & 4 & 7 & 2 & 8 & 0 \\ 0 & 1 & 6 & 8 & 4 & 6 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|cccc} 1 & 2 & 3 & 5 & 7 & 6 & 3 \\ 0 & 1 & 6 & 8 & 4 & 6 & 1 \\ 0 & 0 & 2 & 1 & 8 & 3 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 5 & 6 & 7 & 8 \\ 0 & 0 & 1 & 2 & 4 & 6 & 8 \end{array}\right]$$

Problem 4 Reformulating optimization problems (10 credits)

We consider the max-flow problem on a hypergraph $G = (V, H)$ in the lossy hyperarc model as stated in (2). Let A denote the set of arcs induced by (V, H) and $\mathbb{R}_+$ the set of non-negative real numbers.
As introduced in class and tutorials, $\boldsymbol{x}$ denotes the flow vector, $\boldsymbol{M}$ the incidence matrix describing the induced graph, $\boldsymbol{d}$ the demand vector, $\boldsymbol{N}$ the hyperarc-arc incidence matrix, $\boldsymbol{y}$ the broadcast capacity vector, and $\mathcal{Y}$ the broadcast capacity region.

$$\max r \quad \text{s.t.} \quad \begin{aligned} \boldsymbol{x} &\geq \boldsymbol{0} \\ \boldsymbol{Mx} &= r\boldsymbol{d} \\ \boldsymbol{Nx} &\leq \boldsymbol{y} \\ \boldsymbol{y} &\in \mathcal{Y} \end{aligned} \tag{2}$$

We want to reformulate this optimization problem to a form suitable for Matlab:

$$\min \boldsymbol{f}^T\xi \quad \text{s.t.} \quad \begin{aligned} \boldsymbol{A}\xi &\leq \boldsymbol{b} \\ \boldsymbol{A}'\xi &= \boldsymbol{b}' \\ \boldsymbol{c} \leq \xi &\leq \boldsymbol{c}' \end{aligned} \tag{3}$$

The $\in$ (element of) relation cannot directly be represented by the canonical form given in (3). We therefore introduce the vector $\boldsymbol{\tau}$ of time shares as well as the constraints $\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$ and $\boldsymbol{\tau} \geq \boldsymbol{0}$, which are implied by the broadcast capacity region $\mathcal{Y}$.

4.1* Find a matrix $\boldsymbol{Y} \in \mathbb{R}^{|H|\times|V|}$ such that $\boldsymbol{y} \leq \boldsymbol{Y\tau}$ becomes an equivalent constraint to $\boldsymbol{y} \in \mathcal{Y}$.
Hint: Elements Y_{jv} of $\boldsymbol{Y}$ represent probabilities.

0 1 2

$$Y_{jv} = \begin{cases} 1 - \prod_{k \in A_j} \epsilon_k \text{ or } y_j/\tau_v & \text{if } (v, B) \equiv j \\ 0 & \text{otherwise} \end{cases}$$

For the reformulation we collect all optimization variables into a single vector, which is defined as $\xi = [\boldsymbol{x}\ \boldsymbol{\tau}\ r]^T$.

4.2* Rewrite the equality constraint $\boldsymbol{Mx} = r\boldsymbol{d}$ to the form $[...]\xi = \boldsymbol{0}$.

0 1

$$\begin{aligned} \boldsymbol{Mx} &= r\boldsymbol{d} \\ \boldsymbol{Mx} - r\boldsymbol{d} &= \boldsymbol{0} \\ [\boldsymbol{M} \quad \boldsymbol{0} \quad -\boldsymbol{d}]\xi &= \boldsymbol{0} \end{aligned}$$

4.3 State the dimension of ξ.

0 1

$$\dim \xi = |A| + |V| + 1$$

4.4* Rewrite the inequality constraint $\boldsymbol{N}\boldsymbol{x} \leq \boldsymbol{Y}\boldsymbol{\tau}$ to the form $[...]\boldsymbol{\xi} \leq \boldsymbol{0}$.

$$\boldsymbol{N}\boldsymbol{x} \leq \boldsymbol{Y}\boldsymbol{\tau}$$
$$\boldsymbol{N}\boldsymbol{x} - \boldsymbol{Y}\boldsymbol{\tau} \leq \boldsymbol{0}$$
$$[\boldsymbol{N} \quad -\boldsymbol{Y} \quad \boldsymbol{0}]\boldsymbol{\xi} \leq \boldsymbol{0}$$

4.5 State the dimension of $\boldsymbol{N}$.

$$\dim \boldsymbol{N} = |H| \times |A|$$

4.6* Rewrite the inequality constraint $\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$ to the form $[...]\boldsymbol{\xi} \leq 1$.

$$\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$$
$$[\boldsymbol{0} \quad \boldsymbol{1}^T \quad 0]\boldsymbol{\xi} \leq 1$$

4.7 Combine the results from Subproblems 4.2 – 4.6 to the canonical flow problem as given in (3).

$$\min \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ -1 \end{bmatrix}^T \boldsymbol{\xi} \qquad \text{s. t.} \quad \begin{bmatrix} \boldsymbol{N} & -\boldsymbol{Y} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1}^T & 0 \end{bmatrix} \boldsymbol{\xi} \leq \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} & -\boldsymbol{d} \end{bmatrix} \boldsymbol{\xi} = \boldsymbol{0}$$

$$\boldsymbol{\xi} \geq \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ 0 \end{bmatrix}$$

Sample Solution

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.