Chair of Network Architectures and Services
Department of Informatics
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Eexam
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- During the attendance check a sticker containing a unique code will be put on this exam.
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Network Coding

Exam:	IN2315 / Endterm	**Date:**	Friday 22nd February, 2019
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	13:30 – 15:00

	P 1	P 2	P 3	P 4
I				
II				

Working instructions

- This exam consists of
  - **16 pages** with a total of **4 problems** and
  - a two-sided printed **cheat sheet**.

  Please make sure now that you received a complete copy of the exam.
- Detaching pages from the exam is prohibited.
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- The total amount of achievable credits in this exam is 60 credits.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
  - one **self-written cheat sheet** (A4 double-sided)
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________ to ________ / Early submission at ________

Problem 1 Link quality estimation and estimation of redundancy (10 credits)

In this problem we start with the well-known ETX metric introduced in the lecture and develop advanced link quality estimators. We limit the problem to a two-node wireless network with symmetric channel conditions. For sake of simplicity we assume independent and uniformely distrubted packet loss with probability $0 < \epsilon < 1$. Throughout the problem you may assume a generation size of $N > 0$ packets being transmitted from one node to the other.

1.1* State the number n' of frames to be transferred from one node to the other according to the ETX metric.

1.2* Explain the problem arising from transmitting n' frames, in particular regarding link establishment such as ARP request / response or TCP-handshakes.

1.3* Determine the probability that transmitting n packets allows to decode a generation of N. You may assume that there are no random linear dependencies in transmitted packets.

In order to remedy the situation in Subproblem 1.2, we seek a metric that allows to specify a confidence factor $0 < \sigma < 1$ that transmitting n frames results in a decoding probability of at least σ.

1.4 Based on Subproblem 1.3, state the optimization problem that determines the minimum number n^* of packets required to achieve a decoding probability of at least σ.

1.5 Assuming we choose $\sigma = 0.99$, briefly discuss the effect on a link establishment phase as described above, and on a bulk data transfer.

0 1 2

1.6 In the lecture we introduced RALQE. Discuss how it prevents the situation of Subproblem 1.5.

0 1 2

Problem 2 IEEE 802.11 medium access (19 credits)

This problem discusses the distributed coordination function (DCF), which is the basic medium access strategy of IEEE 802.11-based networks. The DCF is schematically depicted in Figure 2.1.

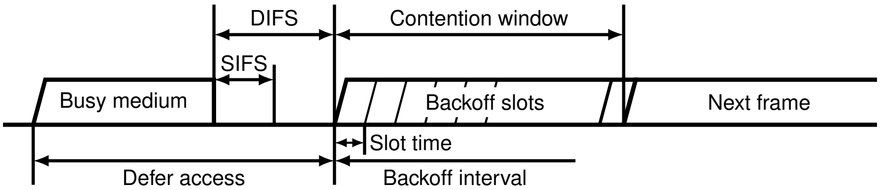


Figure 2.1: IEEE 802.11 medium access

2.1* Explain how the DCF works when a node is ready to transmit a frame (assuming no prior frame loss).

2.2* How is frame loss detected in case of unicasts and multicasts?

2.3* Explain whether or not transmitting nodes are able to differentiate between frame loss and collisions.

2.4* Explain whether or not the DCF is fully functional in case of nodes operating in monitor mode.

0
1
2

We now assume a network consisting of two nodes operating in monitor mode in range of each other. For the sake of simplicity we assume that

- both nodes are backlogged,
- no further communication of other nodes takes place,
- no random frame losses occur, and
- both nodes are perfectly synchronized, i. e., time is slotted and both nodes have a common view of when a time slot starts.

Let $X_i \in \{0, 1, ..., N_i\}$ denote the random variable denoting the number of contention slots drawn by node $i \in \{1, 2\}$.

2.5* Determine the expectation $\mathbb{E}[X_i]$ and briefly discuss its influence on the expected maximum throughput.

0
1
2

2.6* Derive the probability of a collision in case of $N_1 = N_2 = N$.

0
1

0
1
2

2.7* Derive the probability of a collision in case of $N_1 < N_2$.

0
1
2
3
4

2.8* Derive the probability that node 2 successfully transmits a frame in that case.

Problem 3 **Random Linear Network Coding (21 credits)**

We consider extension fields of order $q = p^n$, i. e., sets of polynomials

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \mid a_i \in \mathbb{F}_p \right\}.$$

Elements of such an extension field can also be denoted in short form as $f(x) \in F_q[x] \equiv f(p)$, i. e.,

$$f(x) = x^2 + 2x + 1 \in F_{5^3}[x] \equiv f(5) = 5^2 + 2 \cdot 5 + 1 = 31. \tag{1}$$

3.1* Which conditions must p and n fulfill such that $F_q[x]$ becomes an extension field?

0
1

We now consider $p = 3$, i. e., we have coefficients $a_i \in \mathbb{F}_3$.

3.2* State the summation and multiplication tables for $\mathbb{F}_3$.

0
1

We now construct an extension field of order $q = 9$, i. e., $p = 3$ and $n = 2$.

3.3* List all elements of $F_9[x]$.

0
1

0 1 2 3 4

3.4 Find a reduction polynomial for $F_9[x]$.

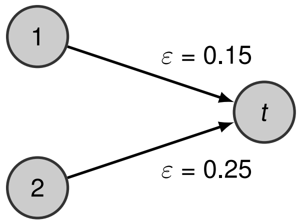
0 1 2 3 4 5 6

3.5 State the summation and multiplication tables for F_9.

In Figure 3.1 we see a fragment of a network. Node t is the destination of some flow. The network coding uses the finite field F_9 with a generation size of 3. The coding vectors and (coded) packets known to each node are depicted next to the respective node.

$$\boldsymbol{C}_1 = \left[\begin{array}{ccc|cccc} 1 & 3 & 2 & 7 & 8 & 1 & 3 \\ 2 & 2 & 3 & 3 & 6 & 0 & 7 \end{array}\right]$$

$$\boldsymbol{C}_2 = \left[\begin{array}{ccc|cccc} 2 & 1 & 6 & 7 & 5 & 3 & 6 \\ 1 & 2 & 3 & 1 & 7 & 1 & 1 \end{array}\right]$$



$$\boldsymbol{C}_t = \left[\begin{array}{ccc|cccc} 1 & 2 & 3 & 5 & 7 & 6 & 3 \end{array}\right]$$

Figure 3.1: Fragment of a Network

Node t has already received one packet but still needs at least two more packets to be able to decode. We want to maximize the probability that an innovative packet is received when a packet is sent.

3.6 Argue which node should transmit a packet next.

3.7 Determine the probability that an innovative packet is received if node 2 transmits a packet.

0
1
2
3
4

3.8 Assuming that two linear independent packets were received by node t, decode the packets.

Problem 4 Reformulating optimization problems (10 credits)

We consider the max-flow problem on a hypergraph $G = (V, H)$ in the lossy hyperarc model as stated in (2). Let A denote the set of arcs induced by (V, H) and $\mathbb{R}_+$ the set of non-negative real numbers.
As introduced in class and tutorials, $\boldsymbol{x}$ denotes the flow vector, $\boldsymbol{M}$ the incidence matrix describing the induced graph, $\boldsymbol{d}$ the demand vector, $\boldsymbol{N}$ the hyperarc-arc incidence matrix, $\boldsymbol{y}$ the broadcast capacity vector, and $\mathcal{Y}$ the broadcast capacity region.

$$\begin{aligned} \max r \quad \text{s. t.} \quad \boldsymbol{x} &\geq \boldsymbol{0} \\ \boldsymbol{Mx} &= r\boldsymbol{d} \\ \boldsymbol{Nx} &\leq \boldsymbol{y} \\ \boldsymbol{y} &\in \mathcal{Y} \end{aligned} \tag{2}$$

We want to reformulate this optimization problem to a form suitable for Matlab:

$$\begin{aligned} \min \boldsymbol{f}^T\boldsymbol{\xi} \quad \text{s. t.} \quad \boldsymbol{A\xi} &\leq \boldsymbol{b} \\ \boldsymbol{A}'\boldsymbol{\xi} &= \boldsymbol{b}' \\ \boldsymbol{c} \leq \boldsymbol{\xi} &\leq \boldsymbol{c}' \end{aligned} \tag{3}$$

The $\in$ (element of) relation cannot directly be represented by the canonical form given in (3). We therefore introduce the vector $\boldsymbol{\tau}$ of time shares as well as the constraints $\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$ and $\boldsymbol{\tau} \geq \boldsymbol{0}$, which are implied by the broadcast capacity region $\mathcal{Y}$.

4.1* Find a matrix $\boldsymbol{Y} \in \mathbb{R}^{|H|\times|V|}$ such that $\boldsymbol{y} \leq \boldsymbol{Y\tau}$ becomes an equivalent constraint to $\boldsymbol{y} \in \mathcal{Y}$. 0 1 2
Hint: Elements Y_{jv} of $\boldsymbol{Y}$ represent probabilities.

For the reformulation we collect all optimization variables into a single vector, which is defined as $\boldsymbol{\xi} = [\boldsymbol{x}\ \boldsymbol{\tau}\ r]^T$.

4.2* Rewrite the equality constraint $\boldsymbol{Mx} = r\boldsymbol{d}$ to the form $[...]\boldsymbol{\xi} = \boldsymbol{0}$. 0 1

4.3 State the dimension of $\boldsymbol{\xi}$. 0 1

4.4* Rewrite the inequality constraint $\boldsymbol{N}\boldsymbol{x} \leq \boldsymbol{Y}\boldsymbol{\tau}$ to the form $[...]\boldsymbol{\xi} \leq \mathbf{0}$.

4.5 State the dimension of $\boldsymbol{N}$.

4.6* Rewrite the inequality constraint $\mathbf{1}^T\boldsymbol{\tau} \leq 1$ to the form $[...]\boldsymbol{\xi} \leq 1$.

4.7 Combine the results from Subproblems 4.2 – 4.6 to the canonical flow problem as given in (3).

$$\min \begin{bmatrix} \quad \end{bmatrix}^T \boldsymbol{\xi} \qquad \text{s.t.} \quad \begin{bmatrix} \qquad \end{bmatrix} \boldsymbol{\xi} \leq \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} \qquad \end{bmatrix} \boldsymbol{\xi} = \mathbf{0}$$

$$\boldsymbol{\xi} \geq \begin{bmatrix} \quad \end{bmatrix}$$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.