Chair of Network Architectures and Services Department of Informatics Technical University of Munich



#### **Eexam**

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#### Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
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#### **Network Coding**

**Exam:** IN2315 / Endterm **Date:** Tuesday 18<sup>th</sup> February, 2020

**Examiner:** Prof. Dr.-lng. Georg Carle **Time:** 13:30 – 15:00

#### Working instructions

- This exam consists of 12 pages with a total of 5 problems.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- · Detaching pages from the exam is prohibited.
- · Allowed resources:
  - one cheatsheet (A4)
  - one non-programmable pocket calculator
  - one analog dictionary English ↔ native language
- Subproblems marked by \* can be solved without results of previous subproblems.
- Answers are only accepted if the solution approach is documented. Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- · Do not write with red or green colors nor use pencils.
- · Physically turn off all electronic devices, put them into your bag and close the bag.

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## Problem 1 Network flow problem (22 credits)

We consider the four-node wireless network G = (N, A), where all possible induces arcs are depicted in Figure 1.1.

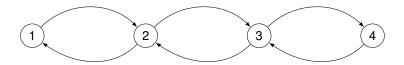


Figure 1.1: four-node network

We assume that packet losses, i. e., erasure events, are independently and identically distributed with expectation  $\epsilon_k$  for all  $k \in A$ . Assume that all arcs  $k \in A$  have unit capacity. Resource shares are denoted by  $0 \le \tau_i \le 1$  for all  $i \in N$ . We further assume orthogonal medium access, i. e., nodes do not transmit concurrently. All nodes are in range of each other, i. e., only one node can transmit at a time.

0	a)* Enumerate the arcs in Figure 1.1 in lexicographic order as known from the lecture.
1/2	b)* Determine the incidence matrix.
0 1	
0	c)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic ascending order and assign numbers $j \equiv (a, B)$ in Table 1.1.
2	d)* List the set of induced arcs $A_j$ for all $j \in H$ in Table 1.1.
0	e)* Determine the network's hyperarc capacity region (Table 1.1).
2	f) Determine the network's broadcast capacity region (Table 1.1). We now consider a unicast session between nodes $s=1$ and $t=4$ .
0	g)* Enumerate all $s - t$ cuts $S$ and their respective capacities $v(S_a)$ .
2 3	cut capacity
0	
¹ <b>  </b>	
3	
0	

$(a,B)\in\mathcal{H}$	j ≡ (a, B)	$A_j$	Z <sub>j</sub>	$y_j$

Table 1.1: Fill in values from different subproblems. (An additional pre-print can be found on Page 12)

0 1	h)* Write the link quality condition(s) for the communication from s to t.
0	i)* Express the minimum number of packets to be transmitted by each node on average to transmit <i>N</i> packets.
0 1 2	j) Express the resource shares considering all nodes in range.
3	
o <b></b>	k) Express the resource shares considering nodes 1 and 3 are not in range of each other.
1 2 3	

## Problem 2 Finite fields and finite extension fields (12 credits)

Given a finite field (Galois field)  $\mathbb{F}_p \subset \mathbb{N}_0$  as introduced in the lecture, we seek to define a set of polynomials  $F_a[x]$  such that  $F_a[x]$  becomes a finite extension field over  $\mathbb{F}_p$ .

a)* State the condition on $p \in \mathbb{N}$ as well as the finite operators $<+,\cdot>$ such that $\mathbb{F}_p$ is a finite field.	_ <b>H</b>
b)* For which $q$ is an extension field guaranteed to exist?	
b) For which q is an extension held guaranteed to exist?	
c) State the set of elements $F_q[x]$ of an extension field over $\mathbb{F}_p$ .	
d) Define the binary operators $<+,\cdot>$ such that $F_q[x]$ becomes an extension field. <b>Note:</b> Take care to fully define all variables you use in your definition! For any $a,b\in F_q[x]$ must hold $a+b:==\sum_{i=0}^{n-1}\left(a_i+b_i \mod p\right)x^i$	
$a \cdot b :=$	

	There are various different implementations for multiplication over binary extension fields. Common approaches are <i>full table lookups</i> and <i>log tables</i> .
0	e) Briefly explain how <i>full table lookups</i> work.
0 1 2 2	f) Briefly explain how <i>log table lookups</i> work.
0 1 2 2	g) For each of those algorithms, give an example of a binary extension field for which you would use the respective algorithm and explain, why you think this algorithm is appropriate for that field.

#### Problem 3 Link quality estimations (12 credits)

A receiver receives eight packets with the following sequence numbers:

$$s_1 = 0$$

$$s_2 = 22$$

$$s_3 = 68$$

$$s_4 = 98$$

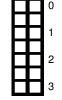
$$s_5 = 102$$

$$s_6 = 126$$

$$s_3 = 68$$
  
 $s_7 = 152$ 

$$s_8 = 201.$$

a)\* Calculate the number of accumulated received  $(p_k)$ , accumulated lost  $(q_k)$ , and lost-since-the-lastreception packets  $(z_k)$ .



$$p_1 =$$

$$p_5 =$$

$$p_5 = q_1 =$$

$$q_1 =$$

$$q_5 = z_1 = z_1$$

$$p_2 =$$

$$p_6 = q_2 = q_2 = q_3$$

$$p_3 =$$

$$p_7 = q_3 = q_3 = q_3$$

$$q_3 =$$

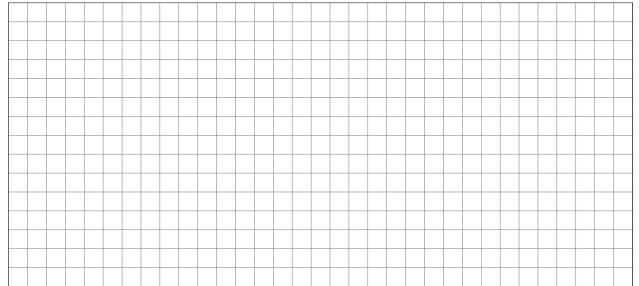
$$q_7 =$$

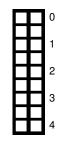
$$z_3 =$$

$$q_4 =$$

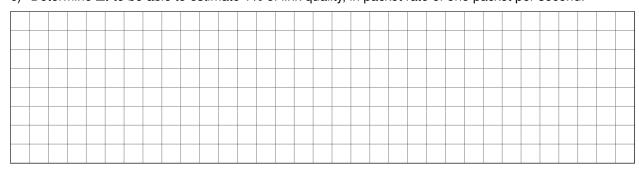
$$q_8 = z_4 =$$

b) Calculate and update EWMA link quality estimation after every reception for given  $\alpha$  = 0.97 and initial estimation EWMA[0] = 1.0.





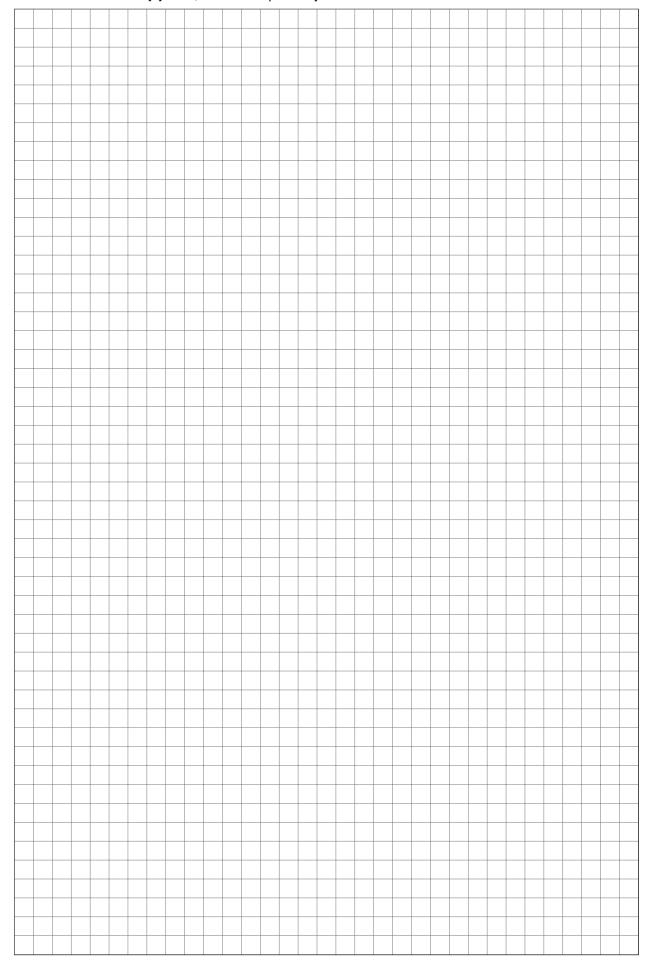
c)\* Determine  $\Delta t$  to be able to estimate 1% of link quality, in packet rate of one packet per second.







d) Calculate and update *WM-EWMA* link quality estimation after every reception for given  $\alpha$  = 0.97, initial estimation *WM-EWMA*[0] = 1.0, and for the previously calculated  $\Delta t$ .



# Problem 4 ETX and EOTX metric (9 credits)

We consider the network depicted in Figure 4.1 that consists of four nodes  $N = \{1, 2, 3, 4\}$  ordered according to their EOTX distance  $d_{ij} \ \forall i, j \in N$  from left to right in ascending order. The erasure probabilities between nodes are considered independently and identically desitrbuted with expectation  $0 < \epsilon_{ij} < 1 \ \forall i, j \in N$ .



2





Figure 4.1: Four-node network. We consider all possible hyperarcs being present.

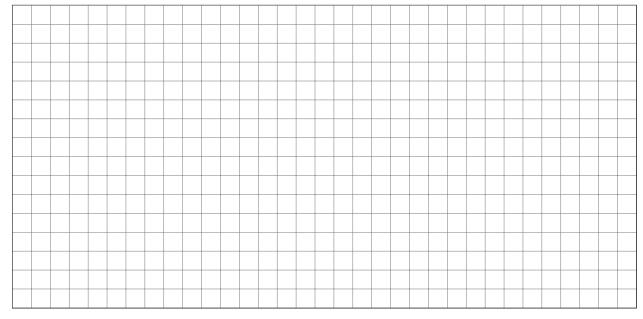
The EOTX distance  $d_{ij}$  between nodes  $i, j \in N$  is known from the lecture to be

$$d_{ij} = \underbrace{\frac{1}{1 - \prod_{k < i} \epsilon_{ik}}}_{a)} + \underbrace{\sum_{k < i} d_{kj} (1 - \epsilon_{ik}) \prod_{l < k} \epsilon_{il}}_{b)}. \tag{1}$$

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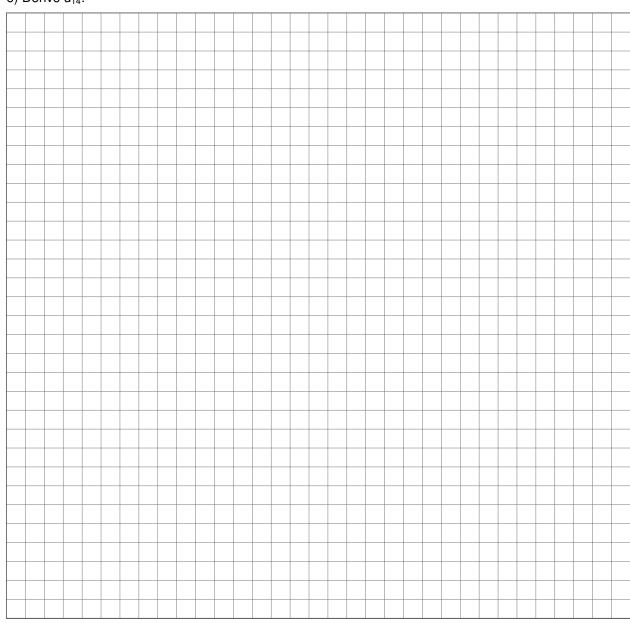


d) Derive  $d_{13}$ .





e) Derive  $d_{14}$ .



# Problem 5 Quiz (5 credits)

Each of the following subproblems can be solved independently of each other. a)\* Explain the difference between network coding and forward error correction. b)\* Give an example of bidirectional network coding. c)\* In multicast networks, state the reasons for maximum flow increase through store-forward, tree-based forwarding and network coding approaches both formally (in terms of constraints) and practically (in terms of actions in the nodes). store-forward vs tree-based forwarding formal: practical: tree-based forwarding vs network coding formal: practical: d)\* How are control frames prioritized over data frames when using the distributed coordination function (DCF) in IEEE 802.11?

$(a,B)\in\mathcal{H}$	j ≡ (a, B)	$A_j$	Z <sub>j</sub>	<b>y</b> <sub>j</sub>

Table 5.1: Additional pre-print for Problem 1