Eexam
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Note:

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# Network Coding 

Exam: IN2315 / Endterm Date: Tuesday 18 ${ }^{\text {th }}$ February, 2020<br>Examiner: Prof. Dr.-Ing. Georg Carle

## Working instructions

- This exam consists of $\mathbf{1 2}$ pages with a total of $\mathbf{5}$ problems. Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
- one cheatsheet (A4)
- one non-programmable pocket calculator
- one analog dictionary English $\leftrightarrow$ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- Answers are only accepted if the solution approach is documented. Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.
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## Problem 1 Network flow problem (22 credits)

We consider the four-node wireless network $G=(N, A)$, where all possible induces arcs are depicted in Figure 1.1.


Figure 1.1: four-node network
We assume that packet losses, i.e., erasure events, are indepedently and identically distributed with expectation $\epsilon_{k}$ for all $k \in A$. Assume that all arcs $k \in A$ have unit capacity. Resource shares are denoted by $0 \leq \tau_{i} \leq 1$ for all $i \in N$. We further assume orthogonal medium access, i.e., nodes do not transmit concurrently. All nodes are in range of each other, i. e., only one node can transmit at a time.
a)* Enumerate the arcs in Figure 1.1 in lexicographic order as known from the lecture.
b)* Determine the incidence matrix.

c)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic ascending order and assign numbers $j \equiv(a, B)$ in Table 1.1.
d) ${ }^{*}$ List the set of induced $\operatorname{arcs} A_{j}$ for all $j \in H$ in Table 1.1.

e)* Determine the network's hyperarc capacity region (Table 1.1).
f) Determine the network's broadcast capacity region (Table 1.1).

We now consider a unicast session between nodes $s=1$ and $t=4$.
$g)^{*}$ Enumerate all $s-t$ cuts $S$ and their respective capacities $v\left(S_{a}\right)$.

|  | cut |
| :--- | :--- |
|  |  |



Table 1.1: Fill in values from different subproblems. (An additional pre-print can be found on Page 12)
$h)^{*}$ Write the link quality condition(s) for the communication from $s$ to $t$.

i)* Express the minimum number of packets to be transmitted by each node on average to transmit $N$ packets.

j) Express the resource shares considering all nodes in range.

k) Express the resource shares considering nodes 1 and 3 are not in range of each other.

## Problem 2 Finite fields and finite extension fields (12 credits)

Given a finite field (Galois field) $\mathbb{F}_{p} \subset \mathbb{N}_{0}$ as introduced in the lecture, we seek to define a set of polynomials $F_{q}[x]$ such that $F_{q}[x]$ becomes a finite extension field over $\mathbb{F}_{p}$.
a) ${ }^{*}$ State the condition on $p \in \mathbb{N}$ as well as the finite operators $<+, \cdot>$ such that $\mathbb{F}_{p}$ is a finite field.

b)* For which $q$ is an extension field guaranteed to exist?

c) State the set of elements $F_{q}[x]$ of an extension field over $\mathbb{F}_{p}$.

d) Define the binary operators $<+, \cdot>$ such that $F_{q}[x]$ becomes an extension field.

Note: Take care to fully define all variables you use in your definition!

For any $a, b \in F_{q}[x]$ must hold...
$a+b:==\sum_{i=0}^{n-1}\left(a_{i}+b_{i} \bmod p\right) x^{i}$
$a \cdot b:=$

There are various different implementations for multiplication over binary extension fields. Common approaches are full table lookups and log tables.
e) Briefly explain how full table lookups work.

f) Briefly explain how log table lookups work.

g) For each of those algorithms, give an example of a binary extension field for which you would use the respective algorithm and explain, why you think this algorithm is appropriate for that field.

## Problem 3 Link quality estimations ( 12 credits)

A receiver receives eight packets with the following sequence numbers:
$s_{1}=0$
$S_{5}=102$
$S_{2}=22$
$s_{6}=126$
$s_{3}=68$
$s_{7}=152$
$s_{4}=98$
$s_{8}=201$.
a)* Calculate the number of accumulated received $\left(p_{k}\right)$, accumulated lost ( $q_{k}$ ), and lost-since-the-lastreception packets $\left(z_{k}\right)$.

| $p_{1}=$ | $p_{2}=$ | $p_{3}=$ | $p_{4}=$ |
| :--- | :--- | :--- | :--- |
| $p_{5}=$ | $p_{6}=$ | $p_{7}=$ | $p_{8}=$ |
| $q_{1}=$ | $q_{2}=$ | $q_{3}=$ | $q_{4}=$ |
| $q_{5}=$ | $q_{6}=$ | $z_{3}=$ | $z_{8}=$ |
| $z_{1}=$ | $z_{2}=$ | $z_{8}=$ |  |
| $z_{5}=$ | $z_{6}=$ |  |  |
|  |  |  |  |

b) Calculate and update EWMA link quality estimation after every reception for given $\alpha=0.97$ and initial estimation $E W M A[0]=1.0$.

c)* Determine $\Delta t$ to be able to estimate $1 \%$ of link quality, in packet rate of one packet per second.

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d) Calculate and update WM-EWMA link quality estimation after every reception for given $\alpha=0.97$, initial estimation $W M-E W M A[0]=1.0$, and for the previosly calculated $\Delta t$.
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## Problem 4 ETX and EOTX metric (9 credits)

We consider the network depicted in Figure 4.1 that consists of four nodes $N=\{1,2,3,4\}$ ordered according to their EOTX distance $d_{i j} \forall i, j \in N$ from left to right in ascending order. The erasure probabilities between nodes are considered independentely and identically desitrbuted with expectation $0<\epsilon_{i j}<1 \forall i, j \in N$.
(1)
(2)
(3)
(4)

Figure 4.1: Four-node network. We consider all possible hyperarcs being present.
The EOTX distance $d_{i j}$ between nodes $i, j \in N$ is known from the lecture to be

$$
\begin{equation*}
d_{i j}=\underbrace{\frac{1}{1-\prod_{k<i} \epsilon_{i k}}}_{a)}+\underbrace{\sum_{k<i} d_{k j}\left(1-\epsilon_{i k}\right) \prod_{1<k} \epsilon_{i l}}_{b)} . \tag{1}
\end{equation*}
$$

a)* Explain in your own words the meaning the of the first summand in (1).

b)* Explain in your own words the meaning the of the second summand in (1).
c) ${ }^{*}$ Derive $d_{12}$.




## Problem 5 Quiz (5 credits)

Each of the following subproblems can be solved independently of each other.
a)* Explain the difference between network coding and forward error correction.
$\square$
b)* Give an example of bidirectional network coding.

c)* In multicast networks, state the reasons for maximum flow increase through store-forward, tree-based forwarding and network coding approaches both formally (in terms of constraints) and practically (in terms of actions in the nodes).

```
store-forward vs tree-based forwarding
formal:
practical:
tree-based forwarding vs network coding
formal:
practical:
```

d)* How are control frames prioritized over data frames when using the distributed coordination function (DCF) in IEEE 802.11?

| $(a, B) \in \mathcal{H}$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $(a, B)$ |  |  |  |  |

Table 5.1: Additional pre-print for Problem 1

