Chair of Network Architectures and Services
Department of Informatics
Technical University of Munich

Esolution
Place student sticker here

Note:
- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Network Coding

Exam:	IN2315 / Endterm	**Date:**	Friday 19th February, 2021
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	14:15 – 15:45

Working instructions

- This exam consists of **12 pages** with a total of **4 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________	to ________	/	Early submission at ________

Problem 1 Finite fields (12 credits)

Given the finite field $\mathbb{F}_p$, we consider the finite extension field

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \;\middle|\; a_i \in \mathbb{F}_p \right\}. \tag{1.1}$$

0
1

a)* State the conditions on p, q, and n such that a finite field $F_q[x]$ exists.

p prime, $n \in \mathbb{N}$, and $q = p^n$

0
1

b)* Reason why there is an extension field for $p = 4$ and $n = 4$.

There is a finite field $\mathbb{F}_4$ with four elements that can be used to create an extension field.

We now consider the binary extension field F_{256} with the reduction polynomial $r(x) = x^8 + x^4 + x^3 + x + 1$, and the two elements $a(x) = x^7 + x + 1$ and $b(x) = x^5 + 1$.

0
1

c)* Determine the product $a(x) \cdot b(x)$ in the given field using polynomial division.

$$a(x) \cdot b(x) = (x^{12} + x^7 + x^6 + x^5 + x + 1) \mod (x^8 + x^4 + x^3 + x + 1) = x^6 + x^3$$

0
1

d)* Discuss the disadvantages of the polynomial division with respect to performance when naively implemented.

Since there may be up to 7 iterations where each intermediate result depends on previous results, the polynomial division cannot be implemented efficiently.

Figure 1.1 shows the log and alog tables for the given field, which is also known from the lecture.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	00	ff	19	01	32	02	1a	c6	4b	c7	1b	68	33	ee	df	03
1	64	04	e0	0e	34	8d	81	ef	4c	71	08	c8	f8	69	1c	c1
2	7d	c2	1d	b5	f9	b9	27	6a	4d	e4	a6	72	9a	c9	09	78
3	65	2f	8a	05	21	0f	e1	24	12	f0	82	45	35	93	da	8e
4	96	8f	db	bd	36	d0	ce	94	13	5c	d2	f1	40	46	83	38
5	66	dd	fd	30	bf	06	8b	62	b3	25	e2	98	22	88	91	10
6	7e	6e	48	c3	a3	b6	1e	42	3a	6b	28	54	fa	85	3d	ba
7	2b	79	0a	15	9b	9f	5e	ca	4e	d4	ac	e5	f3	73	a7	57
8	af	58	a8	50	f4	ea	d6	74	4f	ae	e9	d5	e7	e6	ad	e8
9	2c	d7	75	7a	eb	16	0b	f5	59	cb	5f	b0	9c	a9	51	a0
10	7f	0c	f6	6f	17	c4	49	ec	d8	43	1f	2d	a4	76	7b	b7
11	cc	bb	3e	5a	fb	60	b1	86	3b	52	a1	6c	aa	55	29	9d
12	97	b2	87	90	61	be	dc	fc	bc	95	cf	cd	37	3f	5b	d1
13	53	39	84	3c	41	a2	6d	47	14	2a	9e	5d	56	f2	d3	ab
14	44	11	92	d9	23	20	2e	89	b4	7c	b8	26	77	99	e3	a5
15	67	4a	ed	de	c5	31	fe	18	0d	63	8c	80	c0	f7	70	07

(a) Log

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	01	03	05	0f	11	33	55	ff	1a	2e	72	96	a1	f8	13	35
1	5f	e1	38	48	d8	73	95	a4	f7	02	06	0a	1e	22	66	aa
2	e5	34	5c	e4	37	59	eb	26	6a	be	d9	70	90	ab	e6	31
3	53	f5	04	0c	14	3c	44	cc	4f	d1	68	b8	d3	6e	b2	cd
4	4c	d4	67	a9	e0	3b	4d	d7	62	a6	f1	08	18	28	78	88
5	83	9e	b9	d0	6b	bd	dc	7f	81	98	b3	ce	49	db	76	9a
6	b5	c4	57	f9	10	30	50	f0	0b	1d	27	69	bb	d6	61	a3
7	fe	19	2b	7d	87	92	ad	ec	2f	71	93	ae	e9	20	60	a0
8	fb	16	3a	4e	d2	6d	b7	c2	5d	e7	32	56	fa	15	3f	41
9	c3	5e	e2	3d	47	c9	40	c0	5b	ed	2c	74	9c	bf	da	75
10	9f	ba	d5	64	ac	ef	2a	7e	82	9d	bc	df	7a	8e	89	80
11	9b	b6	c1	58	e8	23	65	af	ea	25	6f	b1	c8	43	c5	54
12	fc	1f	21	63	a5	f4	07	09	1b	2d	77	99	b0	cb	46	ca
13	45	cf	4a	de	79	8b	86	91	a8	e3	3e	42	c6	51	f3	0e
14	12	36	5a	ee	29	7b	8d	8c	8f	8a	85	94	a7	f2	0d	17
15	39	4b	dd	7c	84	97	a2	fd	1c	24	6c	b4	c7	52	f6	01

(b) Alog

Figure 1.1: Log and alog table for GF(256)

e)* Explain the log table approach.

$$a \cdot b = \mathtt{A}[\mathtt{L}[a] + \mathtt{L}[b]]$$

f) Determine the product of $a'(x) = x^2 + x$ and $b'(x) = x^6 + x^4 + x + 1$ using the log table approach.

$$a'(x) = \mathtt{0xdd} \Rightarrow \mathtt{L}[a] = \mathtt{0x1a}$$
$$b'(x) = \mathtt{0x53} \Rightarrow \mathtt{L}[b] = \mathtt{0x30}$$
$$(\mathtt{L}[a] + \mathtt{L}[b]) \mod \mathtt{0xff} = \mathtt{0x4a}$$
$$\mathtt{A}[\mathtt{0x4a}] = \mathtt{0xf1} = x^7 + x^6 + x^5 + x^4 + 1$$

g) State an advantage and a disadvantage of the log table approach with respect to efficiency.

Advantage: result with 3 memory lookups and one addition.
Disadvantage: lookups cannot be parallelized.

Problem 2 Metrics (12 credits)

We consider the wireless network depicted in Figure 2.1 consisting of nodes $N = (s, 1, 2, t)$. Per-node packet erasure probabilities are given $\forall i, j \in N$ as $0 \leq \epsilon_{ij} \leq 1$ and $i \neq j$. Erasures are assumed to be indepentently and identically distributed.

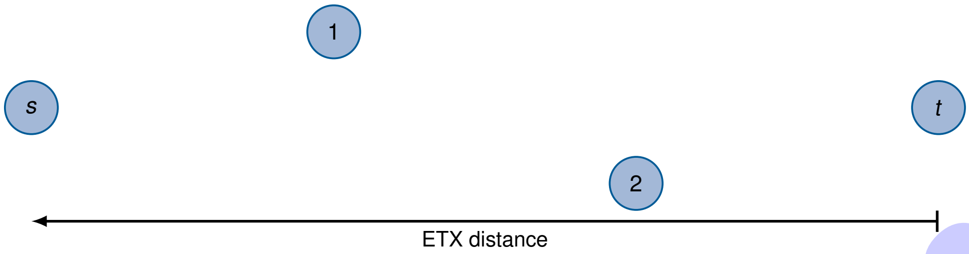


Figure 2.1: Wireless network, all hyperarcs are assumed to exist.

a)* Briefly explain the ETX distance between s and t.

Expected amount of packets transmitted in the network per packet generated at s such that it is received by t.

b)* Argue which distribution the individual terms of the ETX metric adhere to.

Geometric distribution, as it models a waiting problem (retries until success) for independent retries.

In the following, we want to derive the amount of packets individual nodes have to transmit per source packet. To this end, we need the

$$R_j = \sum_{i>j} z_i (1 - \epsilon_{ij}), \tag{2.1}$$

$$L_j = \sum_{i>j} \left(z_i (1 - \epsilon_{ij}) \prod_{k<j} \epsilon_{ik} \right), \text{ and} \tag{2.2}$$

$$z_j = \frac{L_j}{1 - \prod_{k<j} \epsilon_{jk}}. \tag{2.3}$$

c)* Explain R_j as given in (2.1).

R_j is the expected number of packets node i receives from nodes with higher ETX distance per source packet.

d)* Derive R_j for $j \in \{1, 2, t\}$. Note that $R_s = 1$.

$$R_1 = z_s(1 - \epsilon_{s1})$$
$$R_2 = z_s(1 - \epsilon_{s2}) + z_1(1 - \epsilon_{12})$$
$$R_3 = z_s(1 - \epsilon_{st}) + z_1(1 - \epsilon_{1t}) + z_2(1 - \epsilon_{2t})$$

e) Explain L_j as given in (2.2).

L_j is the number of packets node j receives from nodes with higher ETX distance under the condition that no node closer to the destination also receives the packet.

f)* Derive L_j for $j \in N$.

$$L_s = 1 \text{ (per definition)}$$
$$L_t = 0 \text{ (for obvious reasons)}$$
$$L_1 = z_s(1 - \epsilon_{s1})\epsilon_{s2}\epsilon_{st}$$
$$L_2 = z_s(1 - \epsilon_{s2})\epsilon_{st} + z_1(1 - \epsilon_{12})\epsilon_{1t}$$

g)* Explain z_j as given in (2.3).

z_j is the number of packets node j has to transmit per source packet.

h) Derive z_j for $j \in N$.

$$z_t = 0 \text{ (for obvious reasons)}$$
$$z_s = \frac{L_s}{1 - \epsilon_{s1}\epsilon_{s2}\epsilon_{s3}}$$
$$z_1 = \frac{L_1}{1 - \epsilon_{12}\epsilon_{1t}}$$
$$z_2 = \frac{L_2}{1 - \epsilon_{2t}}$$

Problem 3 Network coding in lossy wireless packet networks (22 credits)

We consider the network depicted by the hypergraph $G = (N, \mathcal{H})$ in Figure 3.1. Note that only maximum hyperarcs are drawn.

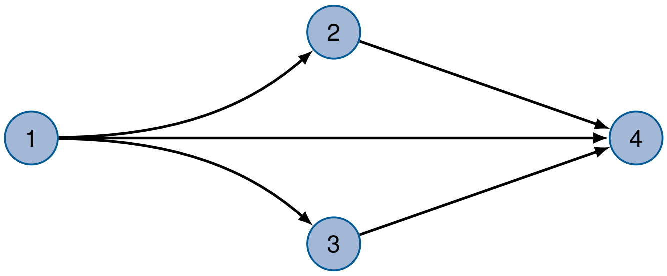
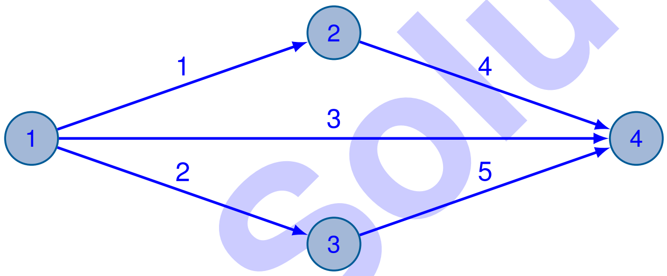


Figure 3.1: Hypergraph of example network, only maximum hyperarcs are drawn

We assume that packet losses, i. e., erasure events, are independently and identically distributed. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume othrogonal medium access, i. e., nodes to not transmit concurrently.

0 1 2

a)* Draw the induced graph $G' = (N, A)$ and number the arcs in lexicographic order.



$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	z_j	y_j
(1,{2})	1	$\tau_1(1-\epsilon_1)\epsilon_2\epsilon_3$	$\tau_1(1-\epsilon_1)$
(1,{3})	2	$\tau_1(1-\epsilon_2)\epsilon_1\epsilon_3$	$\tau_1(1-\epsilon_2)$
(1,{4})	3	$\tau_1(1-\epsilon_3)\epsilon_1\epsilon_2$	$\tau_1(1-\epsilon_3)$
(1,{2,3})	4	$\tau_1(1-\epsilon_1)(1-\epsilon_2)\epsilon_3$	$\tau_1(1-\epsilon_1\epsilon_2)$
(1,{2,4})	5	$\tau_1(1-\epsilon_1)(1-\epsilon_3)\epsilon_2$	$\tau_1(1-\epsilon_1\epsilon_3)$
(1,{3,4})	6	$\tau_1(1-\epsilon_2)(1-\epsilon_3)\epsilon_1$	$\tau_1(1-\epsilon_2\epsilon_3)$
(1,{2,3,4})	7	$\tau_1(1-\epsilon_1)(1-\epsilon_2)(1-\epsilon_3)$	$\tau_1(1-\epsilon_1\epsilon_2\epsilon_3)$
(2,{4})	8	$\tau_1(1-\epsilon_4)$	$\tau_2(1-\epsilon_4)$
(3,{4})	9	$\tau_3(1-\epsilon_5)$	$\tau_3(1-\epsilon_5)$

Table 3.1: Solution table for Problems b) to d)

b)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic order and assign arc indices $j \equiv (a, B)$ in Table 3.1.

c) Determine the network's hyperarc capacity region (Table 3.1).

d) Determine the network's broadcast capacity region (Table 3.1).

We now consider an unicast session between Node 1 and Node 4.

e) List all $s - t$ cuts.

$S_1 = \{1\}$, $S_2 = \{1, 2\}$, $S_3 = \{1, 3\}$, $S_4 = \{1, 2, 3\}$

f) Derive the value of each $s - t$ cut.

$$v(S_1) = y_7 = \sum_{i=1}^{7} z_i = \tau_1(1 - \epsilon_1\epsilon_2\epsilon_3)$$
$$v(S_2) = y_6 + y_8 = z_1 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 = \tau_1(1 - \epsilon_2\epsilon_3) + \tau_2(1 - \epsilon_4)$$
$$v(S_3) = y_5 + y_9 = z_1 + z_2 + z_4 + z_5 + z_6 + z_7 + z_9 = \tau_1(1 - \epsilon_1\epsilon_3) + \tau_3(1 - \epsilon_5)$$
$$v(S_4) = y_3 + y_8 + y_9 = z_3 + z_5 + z_6 + z_7 + z_8 + z_9 = \tau_1(1 - \epsilon_3) + \tau_2(1 - \epsilon_4) + \tau_3(1 - \epsilon_5)$$

We now assume that $\epsilon_5 \geq \epsilon_3 \wedge \epsilon_4 < \epsilon_3 \wedge \epsilon_1 < 1$.

g)* Reason which nodes participate in forwarding traffic.

Node 3 cannot participate since $\epsilon_5 > \epsilon_3$, i. e., even of Node 3 has overheard a packet from Node 1, it is still better for Node 1 to retransmit as it as a higher probability to reach Node 4.

0 1 2

h) Given the conditions above, restate the cut values.

Since $\tau_3 = 0$, the cut values simplifie to:

$$v(S_1) = \sum_{i=1}^{7} z_i = \tau_1(1 - \epsilon_1\epsilon_2\epsilon_3)$$
$$v(S_2) = \tau_1(1 - \epsilon_2\epsilon_3) + \tau_2(1 - \epsilon_4)$$
$$v(S_3) = \tau_1(1 - \epsilon_1\epsilon_3)$$
$$v(S_4) = \tau_1(1 - \epsilon_3) + \tau_2(1 - \epsilon_4)$$

0 1 2

i) Reason which cuts are binding?

We have that $v(S_3) < v(S_1)$ and $v(S_4) < v(S_2)$. Therefore, $v(S_3)$ and $v(S_4)$ are binding.

0 1 2

j) Determine τ_i for all $i \in N$ in that case.

Since $\tau_3 = 0$ we have that $\tau_2 = 1 - \tau_1$

$$v(S_3) = v(S_4)$$
$$\tau_1(1 - \epsilon_1\epsilon_3) = \tau_1(1 - \epsilon_3) + (1 - \tau_1)(1 - \epsilon_4)$$
$$\tau_1 = \frac{1 - \epsilon_4}{1 - \epsilon_4 + \epsilon_3(1 - \epsilon_1)}$$

Problem 4 Quiz (14 credits)

The following subproblems can be solved independently of each other.

a)* Assuming a file is available at three nodes. A fourth node requests the file. Each of the three nodes transmits a random linear combination (uniformely and identically distributed) using XOR only. Determine the decoding probability at the fourth node assuming that no packets are lost.

$$p = \left(1 - \frac{1}{1-2^3}\right)\left(1 - \frac{1}{1-2^2}\right)\left(1 - \frac{1}{1-2^1}\right) \approx 33\,\%$$

b)* The IEEE 802.11 header has (up to) four address fields. Briefly explaint the usage of those fields.

Transmitter, receiver, source, and destination (cmp. link layer routing).

c)* Given a IEEE 802.11-based network. Explain the tradeoff between packet errors and frame size with respect to media access.

Smaller packets decrease error probability. However, media access is most expensive which is why we use rather large packets albeit increasing error probability.

d)* A IEEE 802.11-based network under good conditions has about 2 % packet loss at the PHY. Explain (1) why TCP has problems with such kind of packet loss and (2) why TCP still works fine in that case.

(1) TCP interprets any loss as congestion and reduces TX speed, (2) the MAC layer compensates for losses by using link layer acknowledgements.

e)* Given the incidence matrix $\boldsymbol{M}$ of network. Determine rank$\boldsymbol{M}$.

Number of nodes - 1

0
1

f)* Given the incidence matrix $\boldsymbol{M}$ of network. Explain the intuition behind $\operatorname{rank}\operatorname{null}\boldsymbol{M}$.

Number of linearly independent cycles in a graph.

0
1
2

g)* Given a fully connected wireless network with n nodes. Determine the total amount of possible hyperarcs.

Per node we have up to $2^{n-1} - 1$ hyperarcs and thus $n \cdot (2^{n-1} - 1)$ possible hyperarcs in total.

0
1
2

h) Assuming that the link layer exposes a packet loss rate of 5 % to the network layer. Explain the effect on TCP.

TCP assumes that any segment loss is due to congestion.It would therefore inadvertently decrease its sending window and therefore throtlle data rate.

0
1

i) Describe the hidden station problem.

In a three-node line network, the two outer nodes might be out of range of each other (thus hidden) but may cause collisions at the inner node.

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.