Chair of Network Architectures and Services
School of Computation, Information and Technology
Technical University of Munich

Esolution
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Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Network Coding

Exam:	IN2315 / Endterm	**Date:**	Monday 21st February, 2022
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	14:15 – 15:30

	P 1	P 2	P 3	P 4	P 5
I					
II					

Working instructions

- This exam consists of **12 pages** with a total of **5 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **A4 sheet** with notes
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from __________ to __________ / Early submission at __________

Problem 1 Finite extension fields (12 credits)

Given the finite filed $\mathbb{F}_p$, we consider finite extension fields

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \,\middle|\, a_i \in \mathbb{F}_p \right\}. \tag{1.1}$$

0 1

a)* State the conditions on p, q, and n such that a finite extension field $F_q[x]$ exists.

If $p \in \mathbb{N}$ is prime, $F_q[x]$ exists for any $q = p^n$ with $n \in \mathbb{N}$.

We now consider the finite extension field $F_8[x]$ built upon $\mathbb{F}_2 = \{0, 1\}$.

0 1 2

b)* State two disadvantages of this field with respect to Random Linear Network Coding.

- Elements are represented by 3 bit, which do not fit into the natural word size of computers.
- The probability of two random vectors $\mathbf{x}, \mathbf{y} \in F_2^3[x]$ is rather large (just think about choosing $\mathbf{0}$ as coding vector).

0 1

c)* List all elements of $F_8[x]$.

$$\begin{aligned} F_8[x] = \{&0, 1, \\ &x, x+1, \\ &x^2, x^2+1, x^2+x, x^2+x+1\} \end{aligned}$$

0 1

d)* Explain why a reduction polynomial $r(x)$ is needed for the multiplicative group of $F_q[x]$.

Multiplication of $c = a \cdot b$ of $a, b \in F_q[x]$ may result in $c \notin F_q[x]$.
Therefore, multiplication is defined as

$$c = a \cdot b \bmod r \quad \forall a, b \in F_q[x].$$

e) State the conditions a polyomial $r(x)$ has to fulfill to be a suitable reduction polyomial.

0
1

- $r \neq a \cdot b \ \forall a, b \in F_q[x]$
- $\deg(r) = n + 1$ for $q = 2^n$

A reduction polynomial of $F_8[x]$ is $r(x) = x^3 + x + 1$, which should be used in the following.

f) Find a generator for $F_8[x]$. The solution method must be documented, i. e., only stating the result is not sufficient.

0
1
2
3

$$(x+1)^0 = (x+1)^0 = 1$$
$$(x+1)^1 = (x+1)^1 = x+1$$
$$(x+1)^2 = (x+1)(x+1) = x^2+1$$
$$(x+1)^3 = (x^2+1)(x+1) = x^2$$
$$(x+1)^4 = x^2(x+1) = x^2+x+1$$
$$(x+1)^5 = (x^2+x+1)(x+1) = x$$
$$(x+1)^6 = (x^2+x)(x+1) = x^2+x$$

g) State the Antilog table.

0
1

i	0	1	2	3	4	5	6
A	1	$x+1$	x^2+1	x^2	x^2+x+1	x	x^2+x

h) Calculate the product $(x^2 + x + 1)x^2$ using the log table approach. Note that you can read both log and antilog from the table of Subproblem g). The solution method must be documented, i. e., only stating the result is not sufficient.

0
1
2

$$A\left(L(x^2+x+1) + L(x^2)\right) = A(4+3 \mod 7) = A(0) = 1$$

Problem 2 PTM - libmoep (18 credits)

From lecture and exercises we know the *Packet Transfer Module (PTM)*, a tool based on libmoep. The structure of the PTM is shown in Figure 2.1.

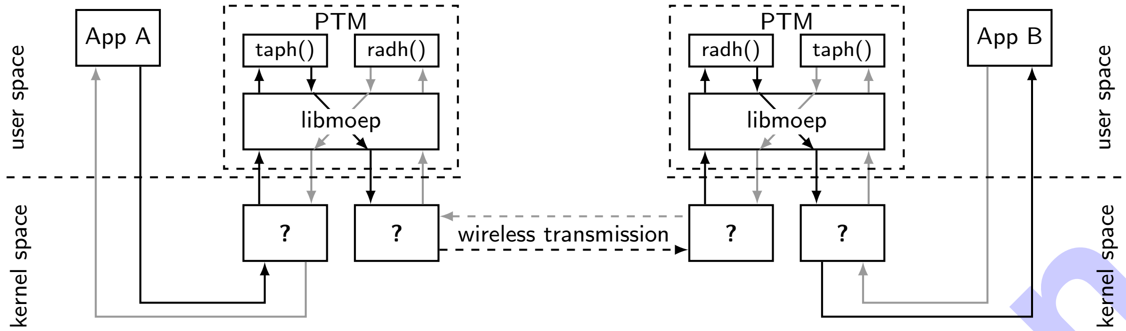


Figure 2.1: Structure of the PTM

a)* Explain the four components of the PTM indicated by a question mark within Figure 2.1. Explain why we need exactly these components.

TAP and MON interfacesTAP interfaces offer networking services to user spaceMON interfaces can be configured with various Wifi settings

App A wants to communicate with App B. IP address and port number under which App B is reachable are already known. We assume that the PTM has just been started on both sides and no other packets have been exchanged so far.

b)* Which is the very first packet (type and purpose) that will be transferred over the wireless link.

ARP request (or neighbor solicitation) to get the MAC address of the TAP interface on the remote host.

c)* Describe the flow of the first packet sent out bei App A throughout all components of the PTM. Explain briefly what happens at each component.

The frame is addressed to the MAC/IP of the remote host and thus by the TAP interface of the local node..
Here, the frame is translated to a moep80211 frame and transferred over the wireless link.
At the remote node, the frame might be received, translated back to IEEE 802.3 and forwarded over the remote TAP interface to the remote application.

d)* Which part is responsible for generating and parsing the radiotap header?

The radh parses the radiotap headerThe function sending the frame is responsible for creating the radiotap handler rad_tx

During the exercises you played around with some radiotap header options. Now it seems that the radiotap header is not generated as intended anymore. You want to find out how the radiotap header looks like at the moment. For this you set up a new computer with a wifi card in monitor mode.

e) Explain why you cannot debug the radiotap header on your newly set up computer.

The radiotap header is used for communication between the hardware and the ptmThe radiotap header is never sent over the air

Assume that App A sends TCP data to App B.

f) How is TCP affected by our PTM implementation?

Packet loss affects TCP as it assumes congestionIn managed mode link layer ACKs result in stable connection. As we are using monitor mode packet loss results in congestions avoidance within TCP

Using libmoep one can easily convert Ethernet frames to frames designed for wireless transmissions. The IEEE 802.11 header lacks a header field that is present within the Ethernet frame.

g) Which header field is missing in the IEEE 802.11 header and how is this issue addressed in libmoep when using the moep802.11 header?

There is no equivalent to the ether type in the MAC802.11 headerFor the moep802.11 header we need to add a pctrl header

The Network Coding Module (NCM) is based on the PTM and uses the same frame format.

h)* Which is the first header field that must be encoded. Explain why it is mandatory to be encoded.

The pctrl field must be encoded. It conains the type of payload and its length. When coding, all packets coded together must be of the same length (padding).To reverse that and to restore the correct type of multiple packets being coded together, this information must be coded alongside that data.

Problem 3 Network coding in lossy wireless packet networks (15 credits)

We consider the network depicted by the hypergraph $G = (N, \mathcal{H})$ in Figure 3.1. **Note that only maximum hyperarcs are drawn**, which imply all smaller ones.

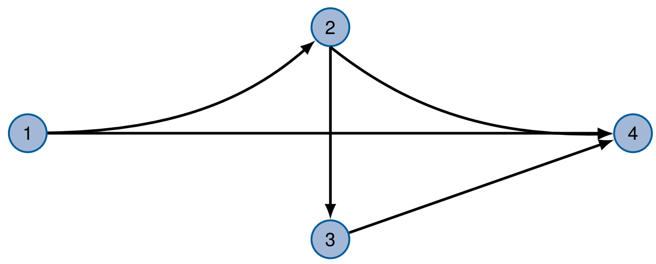
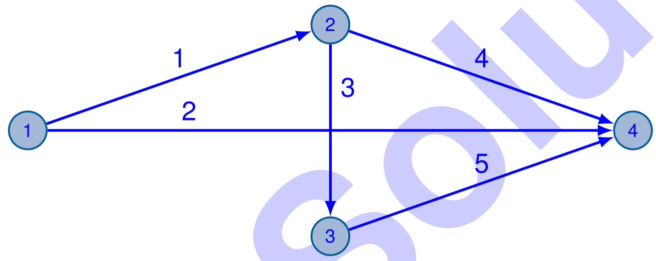


Figure 3.1: Hypergraph of example network, only maximum hyperarcs are drawn

We assume that packet losses, i. e., erasure events, are independently and identically distributed. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume othorgonal medium access, i. e., nodes do not transmit concurrently.

a)* Draw the induced graph $G' = (N, A)$ and number the arcs in lexicographic order.



$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	z_j	y_j
(1,{2})	1	$\tau_1(1-\epsilon_1)\epsilon_2$	$\tau_1(1-\epsilon_1)$
(1,{4})	2	$\tau_1(1-\epsilon_2)\epsilon_1$	$\tau_1(1-\epsilon_2)$
(1,{2,4})	3	$\tau_1(1-\epsilon_1)(1-\epsilon_2)$	$\tau_1(1-\epsilon_1\epsilon_2)$
(2,{3})	4	$\tau_2(1-\epsilon_3)\epsilon_4$	$\tau_2(1-\epsilon_3)$
(2,{4})	5	$\tau_2(1-\epsilon_4)\epsilon_3$	$\tau_2(1-\epsilon_4)$
(2,{3,4})	6	$\tau_2(1-\epsilon_3)(1-\epsilon_4)$	$\tau_2(1-\epsilon_3\epsilon_4)$
(3,{4})	7	$\tau_3(1-\epsilon_5)$	$\tau_3(1-\epsilon_5)$

Table 3.1: Solution table for Problems b) to d)

b)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic order and assign hyperarc indices $j \equiv (a, B)$ in Table 3.1.

c) Determine the network's hyperarc capacity region (Table 3.1).

d) Determine the network's broadcast capacity region (Table 3.1).

We now consider an unicast session between Node 1 and Node 4.

e) List all $s - t$ cuts.

$S_1 = \{1\}$, $S_2 = \{1, 2\}$, $S_3 = \{1, 3\}$, $S_4 = \{1, 2, 3\}$

f) Derive the value of each $s - t$ cut.

$$v(S_1) = y_3 = z_1 + z_2 + z_3$$
$$v(S_2) = y_2 + y_6 = z_2 + z_3 + z_4$$
$$v(S_3) = y_3 + y_7 = z_1 + z_2 + z_3 + z_5$$
$$v(S_4) = y_2 + y_5 + y_7 = z_2 + z_4 + z_5$$

g) Assuming $\epsilon_3 = 1$, explain the condition such that Node 2 can assist in forwarding traffic.

$\epsilon_1 < 1$ (Node 2 needs a chance to receive something from Node 1),and $\epsilon_4 < \epsilon_2$ (otherwise it would be better that Node 1 retransmits).

h) Now consider that $\epsilon_3, \epsilon_5 < 1$. Briefly explain why the condition established in Subproblem g) no longer necessarily holds. (You do not need to state the exact condition, only explain why the previous one is no longer necessary.)

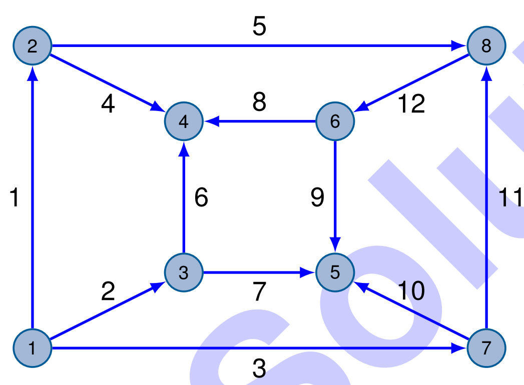
Now, even if $\epsilon_4 > \epsilon_2$ the path over Node 3 may be usefull – depending on ϵ_2, ϵ_3, and ϵ_5.

Problem 4 Multicast Network Models (12 credits)

We consider a so called Hypercube graph $G = (N, A)$ donoted by the incidence matrix

$$\boldsymbol{M} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \tag{4.1}$$

0 1 2 a)* Complete the graph by adding the arcs given within $\boldsymbol{M}$. Number the arcs accordingly.



We are considering a multicast from source $s = \{1\}$ to the terminals $T = \{4, 5\}$. All arcs have unit capacity.

0 ½ b)* Mark source and destination of the multicast within the graph.

0 1 c) Give a multicast tree which contains as few arcs as possible.

(1,3),(3,4),(3,5)

0 1 2 d) What is the highest achievable capacity within the given network using multicast with network coding? (Explanation required.)

3Min-cut is upper bound on multicast rate

0 1 e) Provide the source vector.

$\boldsymbol{d}_{st} = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 \end{bmatrix}^T$

We know the following optimization problem from the lecture.

$$\max \; r \tag{4.2}$$
$$s.t. \quad \boldsymbol{M}\boldsymbol{x}_t = r\boldsymbol{d}_{st} \quad \forall t \in T \tag{4.3}$$
$$\boldsymbol{x}_t \geq \boldsymbol{0} \quad \forall t \in T \tag{4.4}$$
$$\sum_{t \in T} \boldsymbol{x}_t \leq \boldsymbol{z} \tag{4.5}$$

f)* What does the given optimization problem solve?

0 1

max s-T flow / max datarate for the given multicast

g)* Explain the optimization problem using (4.2) – (4.5).

0 1 2

max multicast datarate r
Flow conservation lawfor all terminals
Nonnegativity of flowsfor all terminals
the sum of all commodity flows must not exceed the capacity vector

h)* To which type of multicast treatment does this optimization belong?

0 1

Replicate+ store & forward

i)* Which of the following can be considered multicast communication? (Full credits if answered correctly, no otherwise.)

☒ unicast ☒ bidirectional traffic ☒ broadcast

Problem 5 Short questions (3 credits)

Each of the following Subproblems can be solved independently of each other.

0
1

a)* Briefly explain the difference between the ETX and EoTX metric.

The EoTX metric (Expected Optimal Transmission Count) explicitly considers opportunistic overhearing.

0
1

b)* In which way does FEC differ from Network Coding?

FEC is either done hop by hop or only at source/destination (no recoding at intermediate nodes).

0
1

c)* What is the main difference between CSMA/CD and CSMA/CA?

CSMA/CA always employs a contention window of some minimum size from which a random backoff is drawn before transmission.CSMA/CD does this only when a collision was detected.

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.