Chair of Network Architectures and Services
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Esolution
Sticker will be generated

Compliance to the code of conduct
I hereby assure that I solve and submit this exam myself under my own name by only using the allowed tools listed below.

Signature or full name if no pen input available

Network Coding

Exam:	IN2315 / Endterm Remote	**Date:**	Thursday 23rd February, 2023
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	11:30 – 12:45

Working instructions

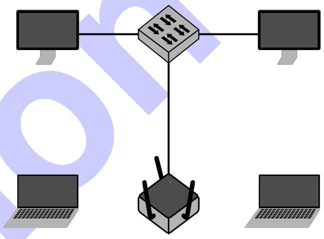
- This exam consists of **12 pages** with a total of **4 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60.5 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one cheatsheet (A4, handwritten, both sides)
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Problem 1 Multiple Choice (12 credits)

The following subproblems are multiple chouce / multiple answer, i. e., at least one answer per subproblem is correct. Subproblems are graded with 1 credit per correct answer and −1 credit per wrong answer. Missing crosses have no influence. The minimal amount of credits per subproblem is 0 credits.

Mark correct answers with a cross ☒
To undo a cross, completely fill out the answer option ■
To re-mark an option, use a human-readable marking ✕■

For Subproblems a) – e) consider the network below, which consists of a wired network with two computers connected to an AP that serves two wireless clients according to IEEE 802.11.



a)* How many broadcast domains does the network contain?

☐ 3 ☐ 6 ☒ 1 ☐ 5 ☐ 2 ☐ 4

b)* How many collision domains does the network contain?

☒ 4 ☐ 2 ☐ 3 ☐ 1 ☐ 6 ☐ 5

c)* Which of the following statements are true?

☒ Computers attached via Ethernet address wireless computers directly.

☐ Wireless computers can differentiate between other wireless clients and computers attached via Ethernet.

☐ Computers attached via Ethernet explicitly address the AP.

☒ Wireless computers address computers attached via Ethernet directly.

☐ Wireless computers commonly bypass the AP when communicating with each other.

☐ Computers attached via Ethernet are aware of the AP.

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d)* Assuming random linear network coding with a generation size of $N \geq 4$, the chance that N packets suffice for decoding ...

☐ increases exponentially with the number of additional coded packets.

☒ primarily depends on the field size.

☐ increases linearly with the number of additional coded packets.

☐ primarily depends on the generation size.

☐ is above 50 % if GF(2) is used.

☒ is roughly 99 % if GF(256) is used.

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e)* Given a network with incidence matrix $\boldsymbol{M} \in \{-1, -0, 1\}^{n \times m}$. Which statements are correct?

☒ $\dim \operatorname{null} \boldsymbol{M} = 1$

☐ $\operatorname{rank} \boldsymbol{M}$ is the number of undirected cycles

☒ $\boldsymbol{M} < n$

☐ $\boldsymbol{M}^{-1}$ exists

---

f)* Which general statements regarding random linear network coding are correct?

☐ Multicast (with replication) can achive higher floes than coding.

☒ For a two-node packet erasure network, coding cannot achieve higher flows than ARQ.

☐ Using pseudo random numbers for coefficients poses a security risk.

☐ The value of a min-cut gives a lower bound for the achievable flow.

☒ The achievable flow is larger than in routed networks.

☐ The flow (according to the min-cut) can always be achieved.

Problem 2 Extension fields (16 credits)

We consider extension fields of order $q = p^n$, i. e., sets of polynomials

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \,\middle|\, a_i \in \mathbb{F}_p \right\}.$$

a) Which conditions must p and n fulfill such that $F_q[x]$ becomes an extension field?

p must be primeand $n \in \mathbb{N}$a natural number.

We now consider $p = 3$, i. e., we have coefficients $a_i \in \mathbb{F}_3 = \{0, 1, 2\}$.

b)* State the summation and multiplication tables for $\mathbb{F}_3$.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

c) State the inverse elements -1 of 1 and -2 of 2 with respect to addition in $\mathbb{F}_3$.

$-1 = 2$

$-2 = 1$

We now construct an extension field of order $q = 9$, i. e., $p = 3$ and $n = 2$.

d)* List all elements of $F_9[x]$.

$$F_9[x] = \{0, 1, 2, x, x+1, x+2, 2x, 2x+1, 2x+2\}$$

e) Explain what a reduction polynomial is.

A polynomial of degree $n+1$ that cannot be represented as product of two elements $a, b \in F[x]$.

f)* Explain what a *primitive element* of $F_9[x]$ is.

A primitive element is an element $a \in F_9[x]$ such that . . . .

For $x^2 + 1$ as reduction polynomial and $x + 2 \in F_9[x]$ as primitive element we get:

$$
\begin{aligned}
(x+2)^0 &= 1 &&=: 01 \qquad & (x+2)^4 &= 2 &&=: 02 \\
(x+2)^1 &= x+2 &&=: 12 & (x+2)^5 &= 2x+1 &&=: 21 \\
(x+2)^2 &= x &&=: 10 & (x+2)^6 &= 2x &&=: 20 \\
(x+2)^3 &= 2x+2 &&=: 22 & (x+2)^7 &= x+1 &&=: 11
\end{aligned}
$$

From these results, we can build the log- and antilog tables:

	0	1	2
0	01	12	10
1	22	02	21
2	20	11	01

(a) L

	0	1	2
0	00	22	11
1	02	21	01
2	20	12	10

(b) A

Table 2.1: Log- and antilog tables for $F_9[x]$

g)* State the Antilog table A in Table 2.1.

h)* State the Log table L in Table 2.1a.

i)* Determine the result of $(x+1)(2x+1)$ using the logtable approach.

$(x+1)(2x+1) = A[L[11] + L[21]] = A[21 + 12] = A[7 + 5 \mod 8]\ A[4] = A[11] = 2 = 2$

j)* Verify your result of Subproblem **??** by using the ordinary polynomial division.
Hint: Do not waste too much time in case the results should differ!

$(x+1)(2x+1) = (2x^2+1) \mod (x^2+1) = 2$

Problem 3 Link layer (15 credits)

We consider the packet erasure network depicted in Figure 3.1. The **bit-error** probability for the link $(1, 2)$ is denoted by ξ. A packet is lost, i. e., erased, if a bit error is detected at the receiving node. For this problem, we assume that all bit-errors are detected.

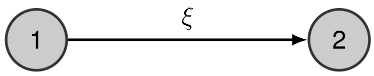

Figure 3.1: Two-node network with **bit-error** probability ξ.

a)* Determine the probability that a frame of length ℓ bit sent by node 1 is correctly received by node 2.

Let X denote the random variable counting the bit-errors within a frame. Then

$$\Pr[X = 0] = (1 - \xi)^{\ell}$$

Now assume that transmissions are protected by some forward error correction code that divides a packet into blocks of k bit, maps those blocks to codewords of length n bit, and allows for the recovery of up to m bit errors within each block.

b)* Determine the probability that a single codeword transmitted by Node 1 is received by Node 2 such that the original block can be recovered.

$$\Pr[Y \leq m] = \sum_{i=0}^{m} \binom{n}{i} (1 - \xi)^{n-i} \xi^{i}$$

c) Determine the packet erasure probability ε, i. e., the probability that a protected frame cannot be recovered.

$$1 - \Pr[Y_1 \leq m, \dots, Y_{\lceil \ell/k \rceil} \leq m] = 1 - \Pr[Y \leq m]^{\lceil \ell/k \rceil} = 1 - \left(\sum_{i=0}^{m} \binom{n}{i} (1 - \xi)^{n-i} \xi^{i} \right)^{\lceil \ell/k \rceil}$$

For some ξ and a specific FEC scheme we obtain a packet erasure probability of $\varepsilon = 0.1$.

d)* Argue why TCP would not perform as expected under those conditions.

TCP interprets any kind of packet (segment) loss as a result of congestion and would thus erroneously initiate its congestion avoidance mechanisms leading to poor throughput.

e) Explain how IEEE 802.11 solves this problem (or at least tries to solve it).

Successfully received frames are acknowledged. If an acknowledgement is not received within a certain time frame, the frame is retransmitted with a more robust encoding (and thus lower speed).

We now assume that packets are transmitted using (unidirectional) random linear network coding (RLNC) with a generation size of N packets. Let the Galois field in use be sufficiently large such that the influence of random linear dependencies can be neglected. RLNC allows the sender to proactively send enough redundant packets such that on average the receiver will be able to decode.

f)* Explain why this is **impossible** without coding and why this is **possible** with coding.

Without coding the sender needs to know beforehand (noncausally) which packets will be lost to proactively retransmit those packets. With coding the sender can proactively transmit redundant coded packets such that the receiver is able to recover any lost packets from the received source and redundant coded packets as long as the receiver receives enough packets in total.

g)* Determine the expected number of packets n^* that have to be sent per generation.

$$n^* = E[X] = \frac{N}{1-\varepsilon} = \frac{N}{0.9}$$

h) Determine the probability that the receiver can decode if n^* packets were transmitted.

$$\Pr[X \geq N] = \sum_{i=N}^{n^*} \binom{n^*}{i} (1-\varepsilon)^i \epsilon^{n^*-i}$$

For $N = 64$ we obtain a probability of $\Pr[X \geq N] = 0.71$.

i)* Explain why this is problematic.

In 30 % sending the expectation is not sufficient. Therefore, retransmitting some packets is necessary, which introduces an additional delay of at least a round-trip-time.

j) Explain how this problem can be solved.

One can compute n^* such that $\Pr[X \geq N] > \theta$ with θ being a confidence level. For e. g. $\theta = 0.99$ one can be sure that sending the respective number of packets n^* is sufficient in 99 %.

Problem 4 Metrics (17.5 credits)

We consider the wireless network depicted in Figure 4.1 consisting of nodes $N = (s, 1, 2, t)$. Per-node packet erasure probabilities are given $\forall i, j \in N$ as $0 \leq \epsilon_{ij} \leq 1$ and $i \neq j$. Erasures are assumed to be indepentently and identically distributed.

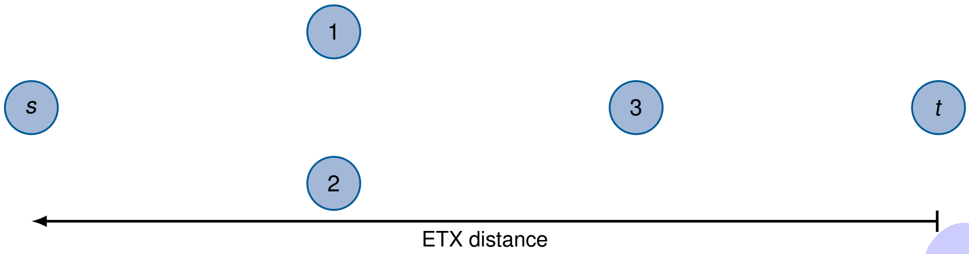

Figure 4.1: Wireless network, all hyperarcs are assumed to exist.

Note that Nodes 1 and 2 have the same ETX distance to the destination.

a)* Briefly explain the ETX distance between s and t.

Expected amount of packets transmitted in the network per packet generated at s such that it is received by t.

b)* Argue which distribution the individual terms of the ETX metric adhere to.

Geometric distribution, as it models a waiting problem (retries until success) for independent retries.

c)* Derive the ETX distance d_{st}^{ETX} as used by MORE.

From the lecture we know that

$$d_{s3}^{\text{ETX}} = \min\left\{\frac{1}{1-\epsilon_{s1}} + \frac{1}{1-\epsilon_{13}}, \frac{1}{1-\epsilon_{s2}} + \frac{1}{1-\epsilon_{23}}\right\}.$$

Since Nodes 2 and 3 are assumed to have the same ETX distance to the destination, both terms in the min-expression are equal.
For the total distance we therefore obtain

$$d_{st}^{\text{ETX}} = d_{s3}^{\text{ETX}} + \frac{1}{1-\epsilon_{34}}.$$

In the following, we consider the E**O**TX metric and want to derive the amount of packets individual nodes have to transmit per source packet. To this end, we need the

$$R_j = \sum_{i>j} z_i(1-\epsilon_{ij}), \tag{4.1}$$

$$L_j = \sum_{i>j} \left(z_i(1-\epsilon_{ij}) \prod_{k<j} \epsilon_{ik} \right), \text{ and} \tag{4.2}$$

$$z_j = \frac{L_j}{1-\prod_{k<j}\epsilon_{jk}}. \tag{4.3}$$

d)* Which factor does the EOTX metric consider that is not considered by the ETX metric? Give a concrete example based on Figure 4.1.

Opportunistic overhearing:
it is decided whether packets should go over Node 1 **or** Node 2. The case that the respective other node could overhear and forward packets is not taken into account.

e)* Explain R_j as given in (4.1).

R_j is the expected number of packets node j receives from nodes with higher ETX distance per source packet.

f)* Derive R_j for $j \in \{1, 2, 3, t\}$. Note that $R_s = 1$.

$R_1 = z_s(1-\epsilon_{s1})$

$R_2 = z_s(1-\epsilon_{s2})$

$R_3 = z_s(1-\epsilon_{s3}) + z_1(1-\epsilon_{13}) + z_2(1-\epsilon_{23})$

$R_t = z_s(1-\epsilon_{st}) + z_1(1-\epsilon_{1t}) + z_2(1-\epsilon_{2t})z_3(1-\epsilon_{3t})$

0 1 2 3

g)* Derive L_j for $j \in N$.

$$
\begin{aligned}
L_s &= 1 \text{ (per definition)} \\
L_t &= 0 \text{ (for obvious reasons)} \\
L_1 &= z_s(1 - \epsilon_{s1})\epsilon_{s3}\epsilon_{st} \\
L_2 &= z_s(1 - \epsilon_{s2})\epsilon_{s3}\epsilon_{st} \\
L_3 &= z_s(1 - \epsilon_{s3})\epsilon_{st} + z_1(1 - \epsilon_{13})\epsilon_{1t} + z_2(1 - \epsilon_{23})\epsilon_{2t}
\end{aligned}
$$

0 1

h)* Explain z_j as given in (4.3).

z_j is the number of packets node j has to transmit per source packet.

0 1 2

i) Derive z_j for $j \in N$.

$$
\begin{aligned}
z_t &= 0 \text{ (for obvious reasons)} \\
z_s &= \frac{L_s}{1 - \epsilon_{s1}\epsilon_{s2}\epsilon_{s3}\epsilon_{st}} \\
z_1 &= \frac{L_1}{1 - \epsilon_{13}\epsilon_{1t}} \\
z_2 &= \frac{L_2}{1 - \epsilon_{23}\epsilon_{2t}} \\
z_3 &= \frac{L_3}{1 - \epsilon_{3t}}
\end{aligned}
$$

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.