Chair of Network Architectures and Services
School of Computation, Information and Technology
Technical University of Munich

Esolution
Place student sticker here

Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Network Coding

Exam:	IN2315 / Endterm	**Date:**	Friday 1st March, 2024
Examiner:	Prof. Dr.-Ing. Stephan Günther	**Time:**	08:00 – 09:15

Working instructions

- This exam consists of **12 pages** with a total of **4 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **page A4 cheatsheet**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from __________ to __________ / Early submission at __________

Problem 1 Multiple choice and short questions (10 credits)

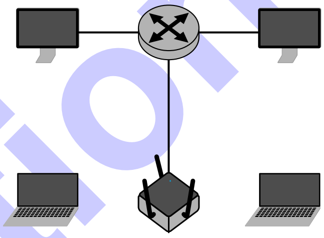
The following subproblems are multiple chouce / multiple answer, i. e., at least one answer per subproblem is correct. Subproblems are graded with 1 credit per correct answer and −1 credit per wrong answer. Missing crosses have no influence. The minimal amount of credits per subproblem is 0 credits.

Mark correct answers with a cross ☒

To undo a cross, completely fill out the answer option ■

To re-mark an option, use a human-readable marking ×■

For Subproblems a) – d) consider the network below, which consists of a wired network with two computers connected to an AP that serves two wireless clients according to IEEE 802.11.



a)* How many broadcast domains does the network contain?

- [x] 3
- [ ] 6
- [ ] 1
- [ ] 5
- [ ] 2
- [ ] 4

b)* How many collision domains does the network contain?

- [x] 4
- [ ] 2
- [ ] 3
- [ ] 1
- [ ] 6
- [ ] 5

c)* Which of the following statements are true?

- [x] Computers attached via Ethernet address wireless computers directly.
- [ ] Wireless computers can differentiate between other wireless clients and computers attached via Ethernet.
- [ ] Computers attached via Ethernet explicitly address the AP.
- [x] Wireless computers address computers attached via Ethernet directly.
- [ ] Wireless computers commonly bypass the AP when communicating with each other.
- [ ] Computers attached via Ethernet are aware of the AP.

---

d)* Assuming random linear network coding with a generation size of $N \geq 4$, the chance that $N+1$ packets suffice for decoding . . .

- [x] is near 100 % for GF(16) and GF(256).
- [ ] primarily depends on the generation size.
- [x] primarily depends on the field size.
- [ ] increases exponentially with the number of additional coded packets.
- [ ] increases linearly with the number of additional coded packets.

---

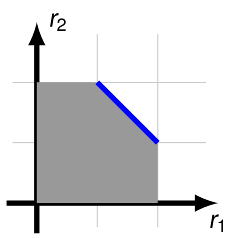
e)* Briefly explain the difference between the ETX and EoTX metric.

The EoTX metric (Expected Optimal Transmission Count) explicitly considers opportunistic overhearing.

f)* In which way does FEC differ from Network Coding?

FEC is either done hop by hop or only at source/destination (no recoding at intermediate nodes).

g)* Given a coded packet network with two flows whose data rates are denoted by r_1, r_2. Its feasable set of solutions is shown in the figure below. Mark the set of solutions maximizing the sum rate.



Problem 2 Finite extension fields (16 credits)

We consider a Galois field $\mathbb{F}_p$. First, answer the following simple questions regarding this finite field.

a)* Given $a, b \in \mathbb{F}_p$, state the rule for the $+$ operation.

$x = (a + b) \mod p$

b)* Given $a, b \in \mathbb{F}_3$, state the rule for the $\cdot$ operation.

$x = (a \cdot b) \mod p$

c)* Which condition must hold for p such that $\mathbb{F}_p$ forms a Galois field.

p must be prime.

We now consider the Galois field formed by $p = 2$. Using this field, we can create so called *finite extension fields* $F_q[x]$.

d)* Briefly explain in your own words what a *finite extension field* is.

Finite extension fields are a set of polynomials with coefficients choosen from the underlying Galois field.

e) State the elements of $F_8[x]$.

$F_{16}[x] = \{0, 1, x, x + 1, x^2, x^2 + 1, x^2 + x, x^2 + x + 1\}$

f) Argue whether $F_8[x]$ can be implemented in an efficient way on today's computers.

Elements of $F_8[x]$ can be represented by groups of three bits.While the coefficients would work fine, they cannot be grouped to multiple of octets which makes it difficult for today's computers.

We still consider $F_8[x]$. A reduction polynomial for this field is $r(x) = x^3 + x + 1$.

0 1

g)* Why do we need a reduction polynomial?

Multiplication of two elements $a, b \in F_8[x]$ might be outside the set. The reduction polynomial forces the result to be within $F_8[x]$.

0 1 2

h) Which general condition(s) must hold for such a reduction polynomial?

It must not be created by any multiplication of $a, b \in F_q[x]$ and must be of power n for $q = p^n$.

0 1 2 3 4

i) Given $a = x^2 + x + 1$ and $b = x^2 + 1$, provide the result of $a \cdot b$ over $F_8[x]$.

$a \cdot b =$ "$x^4 + x^3 + x + 1$"

This has to be reduced by p:

```
11011 : 1011 = 11
1011
----
0110
 1101
 1011
 ----
 0110
```

The result is 110 $= x^2 + x$.

Problem 3 IEEE 802.11 medium access (19 credits)

This problem discusses the distributed coordination function (DCF), which is the basic medium access strategy of IEEE 802.11-based networks. The DCF is schematically depicted in Figure 3.1.

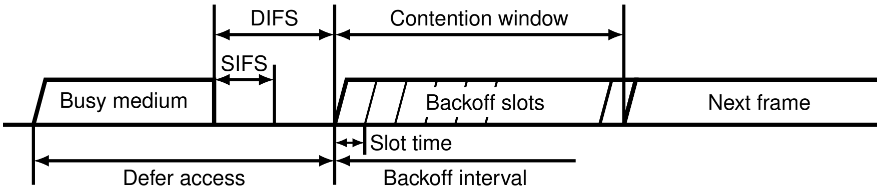


Figure 3.1: IEEE 802.11 medium access

a)* Explain how the DCF works when a node is ready to transmit a frame (assuming no prior frame loss).

After a minimum idle time of DIFS + 2SIFS a random value from the contention window $\{0, 1, \ldots, N\}$ is drawn. Medium access is deferred for that amount of time slots.
If the medium is still idle after that time period, a transmission attempt is made in the following time slot.
If the medium becomes busy in the mean time, transmission and countdown are deferred until the medium becomes idle again.
In case of an unsuccessful transmission, the contention window is exponentially increased up to some maximum value.

b)* How is frame loss detected in case of unicasts and multicasts?

Missing L2-Ack in case of unicasts, no detection possible for multicasts.

c)* Explain whether or not transmitting nodes are able to differentiate between frame loss and collisions.

They are in gernal unable for two reasons:

1. A transmitting node is commonly not able to concurrently sense the medium.
2. Even if a node was able to do that, the second transmission involved into a collision near the receiving node might be out of range (hidden station problem).

d)* Explain whether or not the DCF is fully functional in case of nodes operating in monitor mode.

No, it is not as nodes operating in monitor mode do not transmit L2-ACKs.Therefore, a transmitter is unable to adjust the binary exponential backoff as frame losses cannot be detected.

We now assume a network consisting of two nodes operating in monitor mode in range of each other. For the sake of simplicity we assume that

- both nodes are backlogged,
- no further communication of other nodes takes place,
- no random frame losses occur, and
- both nodes are perfectly synchronized, i. e., time is slotted and both nodes have a common view of when a time slot starts.

Let $X_i \in \{0, 1, \ldots, N_i\}$ denote the random variable denoting the number of contention slots drawn by node $i \in \{1, 2\}$.

e)* Determine the expectation $\mathbb{E}[X_i]$ and briefly discuss its influence on the expected maximum throughput.

$\mathbb{E}[X_i] = (N_i + 1)/2$ as X_i is drawn uniformely from the set $\{0, 1, \ldots, N_i\}$.
The larger $\mathbb{E}[X_i]$ becomes, the less the probability for a collision. However, more time is wasted for medium access resulting in lower maximum throughput.

f)* Derive the probability of a collision in case of $N_1 = N_2 = N$.

$$\Pr[\text{“Collision”}] = \sum_{k=0}^{N} \Pr[X_1 = X_2]$$
$$= \sum_{k=0}^{N} \frac{1}{(N+1)^2} = \frac{1}{N+1}$$

g)* Derive the probability of a collision in case of $N_1 < N_2$.

$$
\begin{aligned}
\Pr[\text{“Collision”}] &= \sum_{k=0}^{\min\{N_1,N_2\}} \Pr[X_1 = X_2] \\
&= \sum_{k=0}^{\min\{N_1,N_2\}} \frac{1}{(N_1+1)(N_2+1)} \\
&= \frac{N_1+1}{(N_1+1)(N_2+1)} = \frac{1}{N_2+1}
\end{aligned}
$$

h)* Derive the probability that node 2 successfully transmits a frame in that case.

$$
\begin{aligned}
\Pr[\text{“Node 2 successfull”}] &= \Pr[X_2 < X_1] \\
&= \sum_{k=0}^{N_1} \Pr[X_2 = k]\Pr[X_1 > k] \\
&= \frac{1}{N_2+1}\sum_{k=0}^{N_1} (1 - \Pr[X_1 \le k]) \\
&= \frac{1}{N_2+1}\sum_{k=0}^{N_1} \left(1 - \frac{k+1}{N_1+1}\right) \\
&= \frac{1}{N_2+1}\left(N_1 + 1 - \frac{1}{N_1+1}\sum_{k=1}^{N_1+1} k\right) \\
&= \frac{1}{N_2+1}\left(N_1 + 1 - \frac{1}{N_1+1}\,\frac{(N_1+1)(N_1+2)}{2}\right) \\
&= \frac{N_1}{2(N_2+1)}
\end{aligned}
$$

Problem 4 Network coding in lossy wireless packet networks (15 credits)

We consider the network depicted by the hypergraph $G = (N, \mathcal{H})$ in Figure 4.1. **Note that only maximum hyperarcs are drawn**, which imply all smaller ones.

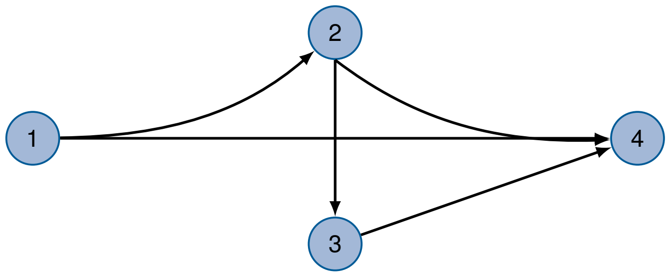
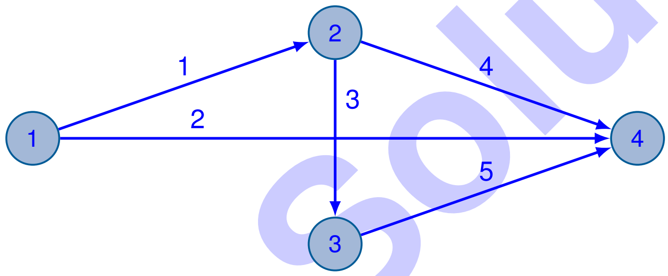


Figure 4.1: Hypergraph of example network, only maximum hyperarcs are drawn

We assume that packet losses, i. e., erasure events, are independently and identically distributed. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume othorgonal medium access, i. e., nodes do not transmit concurrently.

a)* Draw the induced graph $G' = (N, A)$ and number the arcs in lexicographic order.



$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	z_j	y_j
(1,{2})	1	$\tau_1(1-\epsilon_1)\epsilon_2$	$\tau_1(1-\epsilon_1)$
(1,{4})	2	$\tau_1(1-\epsilon_2)\epsilon_1$	$\tau_1(1-\epsilon_2)$
(1,{2,4})	3	$\tau_1(1-\epsilon_1)(1-\epsilon_2)$	$\tau_1(1-\epsilon_1\epsilon_2)$
(2,{3})	4	$\tau_2(1-\epsilon_3)\epsilon_4$	$\tau_2(1-\epsilon_3)$
(2,{4})	5	$\tau_2(1-\epsilon_4)\epsilon_3$	$\tau_2(1-\epsilon_4)$
(2,{3,4})	6	$\tau_2(1-\epsilon_3)(1-\epsilon_4)$	$\tau_2(1-\epsilon_3\epsilon_4)$
(3,{4})	7	$\tau_3(1-\epsilon_5)$	$\tau_3(1-\epsilon_5)$

Table 4.1: Solution table for Problems b) to d)

b)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic order and assign hyperarc indices $j \equiv (a, B)$ in Table 4.1.

c) Determine the network's hyperarc capacity region (Table 4.1).

d) Determine the network's broadcast capacity region (Table 4.1).

We now consider an unicast session between Node 1 and Node 4.

e) List all $s - t$ cuts.

$S_1 = \{1\}$, $S_2 = \{1, 2\}$, $S_3 = \{1, 3\}$, $S_4 = \{1, 2, 3\}$

f) Derive the value of each $s - t$ cut.

$$v(S_1) = y_3 = z_1 + z_2 + z_3$$
$$v(S_2) = y_2 + y_6 = z_2 + z_3 + z_4$$
$$v(S_3) = y_3 + y_7 = z_1 + z_2 + z_3 + z_5$$
$$v(S_4) = y_2 + y_5 + y_7 = z_2 + z_4 + z_5$$

g) Assuming $\epsilon_3 = 1$, explain the condition such that Node 2 can assist in forwarding traffic.

$\epsilon_1 < 1$ (Node 2 needs a chance to receive something from Node 1),and $\epsilon_4 < \epsilon_2$ (otherwise it would be better that Node 1 retransmits).

h) Now consider that $\epsilon_3, \epsilon_5 < 1$. Briefly explain why the condition established in Subproblem g) no longer necessarily holds. (You do not need to state the exact condition, only explain why the previous one is no longer necessary.)

Now, even if $\epsilon_4 > \epsilon_2$ the path over Node 3 may be usefull – depending on ϵ_2, ϵ_3, and ϵ_5.

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.